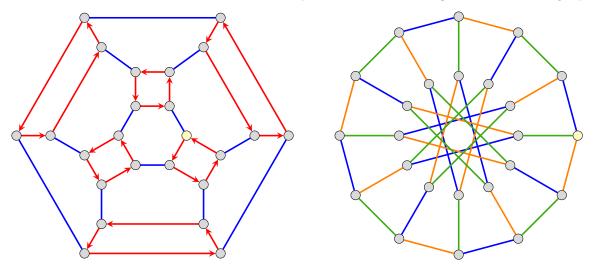
1. Below are two Cayley diagrams for the symmetric group

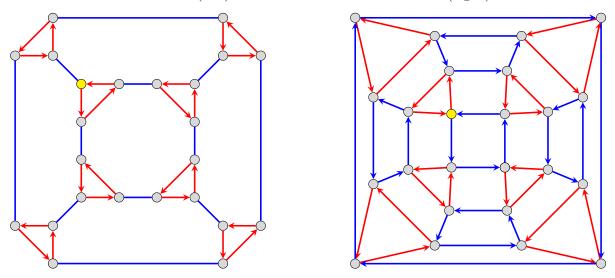
$$S_4 = \langle (1234), (12) \rangle = \langle (12), (13), (14) \rangle.$$

At left is a truncated octahedron, called the *pemutohedron*. At right is the *Nauru graph*.



Carry out the following steps, taking the yellow node to represent the identity.

- (a) On both diagrams, label the nodes by elements of  $S_4$ , written in cycle notation as a product of disjoint cycles.
- (b) On the Nauru graph, label the nodes with permutations of the word **1234**, where  $(i \ j)$  swaps the  $i^{\text{th}}$  and  $j^{\text{th}}$  coordinates.
- (c) On a separate copy of the Nauru graph, label the nodes with permutations of **1234**, where  $(i \ j)$  swaps the numbers i and j.
- 2. Two Cayley diagrams for the symmetric group  $S_4$  arranged on flattened Archimedean solids the truncated cube (left) and the rhombicuboctahedron (right).

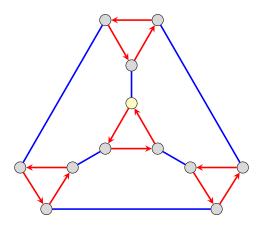


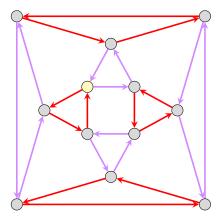
Determine what generating sets will yield these Cayley diagrams. Then, label the nodes with permutations in cycle notation, written as a product of disjoint cycles.

3. The alternating group  $A_4$  is the subgroup of  $S_4$  that consists of the even permutations. Two Cayley diagrams are shown below, for presentations

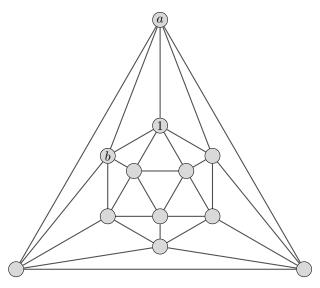
$$A_4 = \langle (123), (12)(34) \rangle = \langle (123), (234) \rangle.$$

Label the nodes of these diagrams with elements of  $A_4$  in cycle notation, written as a product of disjoint cycles.





4. Draw the Cayley diagram of the group  $G = \langle a, b, c \mid a^2 = b^3 = c^3 = abc = 1 \rangle$  on the skeleton of the icosahedron, shown below, and label the nodes with elements written using a, b, and c.



There are five groups of order 12: the abelian groups  $C_{12}$  and  $C_6 \times C_2$ , the dihedral group  $D_6$ , the alternating group  $A_4$ , and the dicyclic group Dic<sub>6</sub>. Determine which group G is isomorphic to, and then re-draw this Cayley diagram with the nodes labeled with elements of that group.

5. Prove that if  $g^2 = e$  for all  $g \in G$ , then G must be abelian.