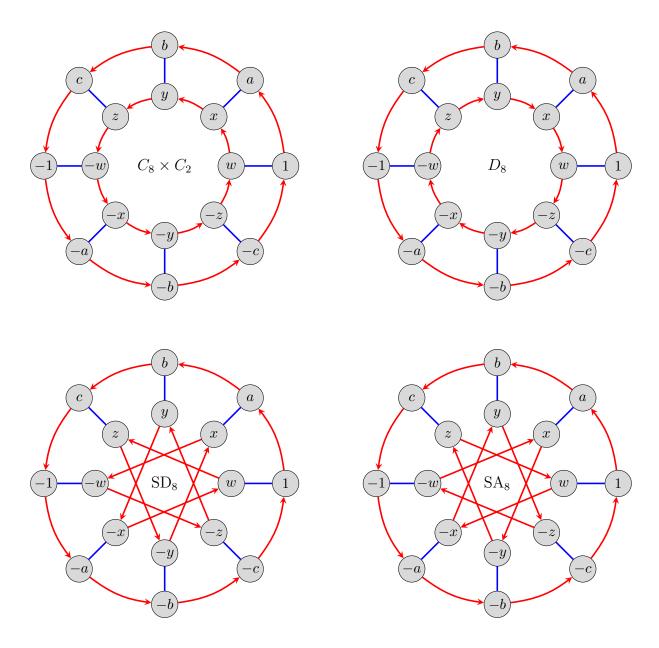
1. Cayley graphs for the four semidirect products of C_8 with C_2 are shown below, with a different labeling scheme on their nodes.



For all four of these groups, identifying each element with its "negative" yields a "quotient group" of order 8, like what we did with the dicyclic group $\text{Dic}_8 = Q_{16}$ in HW 2. Construct a Cayley table and Cayley graph for each of these quotient groups, using the elements

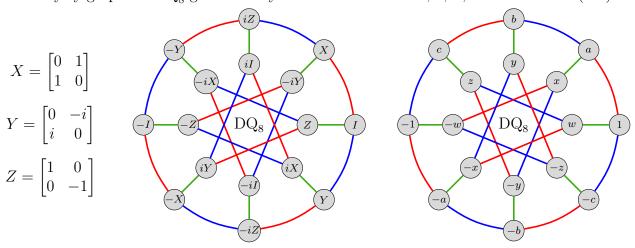
$$\pm 1$$
, $\pm a$, $\pm b$, $\pm c$, $\pm w$, $\pm x$, $\pm y$, $\pm z$,

and determine to which familiar group each is isomorphic. If two groups give the same table and graph, you do not need to write this out twice.

2. The diquaternion group DQ_8 can be constructed from our standard representation of $Q_8 = \langle R_4, S, T \rangle$, along with the reflection matrix F in $D_n = \langle R_n, F \rangle$. That is,

$$\mathrm{DQ}_8 \cong \left\langle i, j, k, f \right\rangle \cong \left\langle \underbrace{\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}}_{R=R_4}, \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}_{S}, \underbrace{\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}}_{T=RS}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{F} \right\rangle$$

A Cayley graph for DQ_8 generated by the *Pauli matrices* X, Y, Z, is shown below (left).



- (a) Find a presentation for $DQ_8 = \langle X, Y, Z \rangle$.
- (b) Construct a Cayley graph for $DQ_8 = \langle R, S, F \rangle$, and find a group presentation. Extra credit will be given for the best-looking construction.
- (c) Carry out the "quotient process" as was done for $C_8 \times C_2$, D_8 , SD_8 , and SA_8 in the previous problem; see the graph on the right.
- 3. For this problem, the use of an online matrix calculator, like https://matrixcalc.org, that can handle complex exponential inputs, is strongly recommended.
 - (a) The dicyclic group Dic_n is only defined when n is even. However, if we try to define

$$Dic_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & \overline{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\rangle,$$

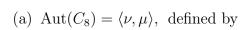
then the result is still a group. Determine which group this is, with justification.

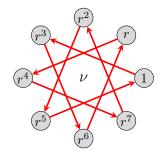
(b) The diquaternion and semidihedral groups, DQ_n and SD_n , are only defined when $n = 2^m$. However, we can still define

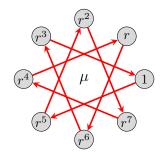
$$\mathrm{DQ}_6 := \left\langle \begin{bmatrix} \zeta_6 & 0 \\ 0 & \overline{\zeta}_6 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle, \qquad \mathrm{SD}_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & -\overline{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle.$$

What groups are these? Construct a Cayley graph for each.

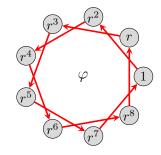
4. The automorphism group of C_n is isomorphic to U_n , the multiplicative group of integers modulo n, from HW 2, #2. For each of the following, construct a Cayley graph of $\operatorname{Aut}(C_n)$ with the nodes labeled by re-wirings, and a Cayley table for this group. Then determine its isomorphism type.



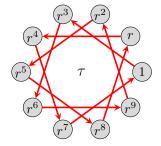




(b) $\operatorname{Aut}(C_9) = \langle \varphi \rangle$, defined by



(c) $\operatorname{Aut}(C_{10}) = \langle \tau \rangle$, defined by



(d) $\operatorname{Aut}(C_{16}) = \langle \rho, \sigma \rangle$, defined by

