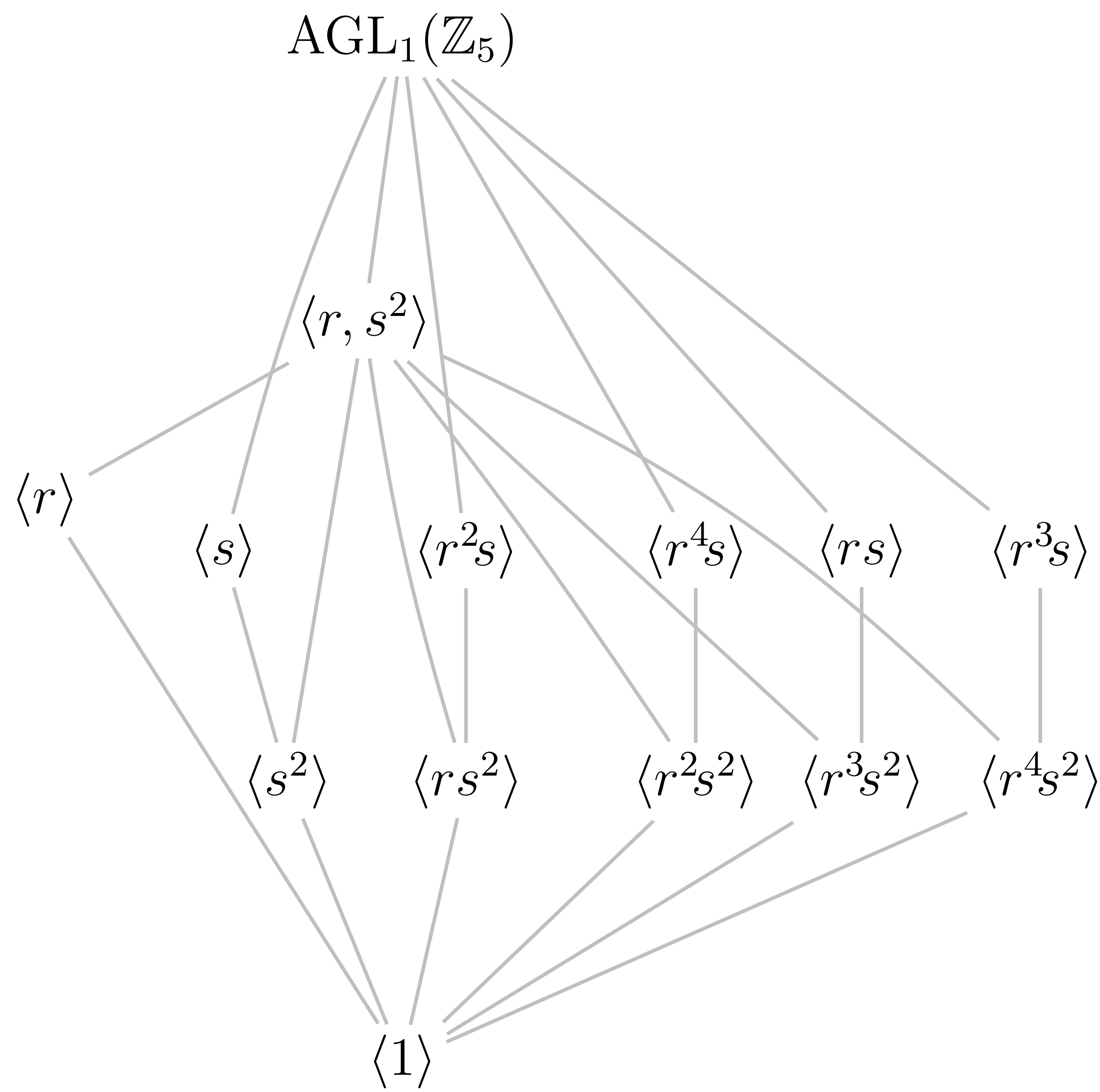
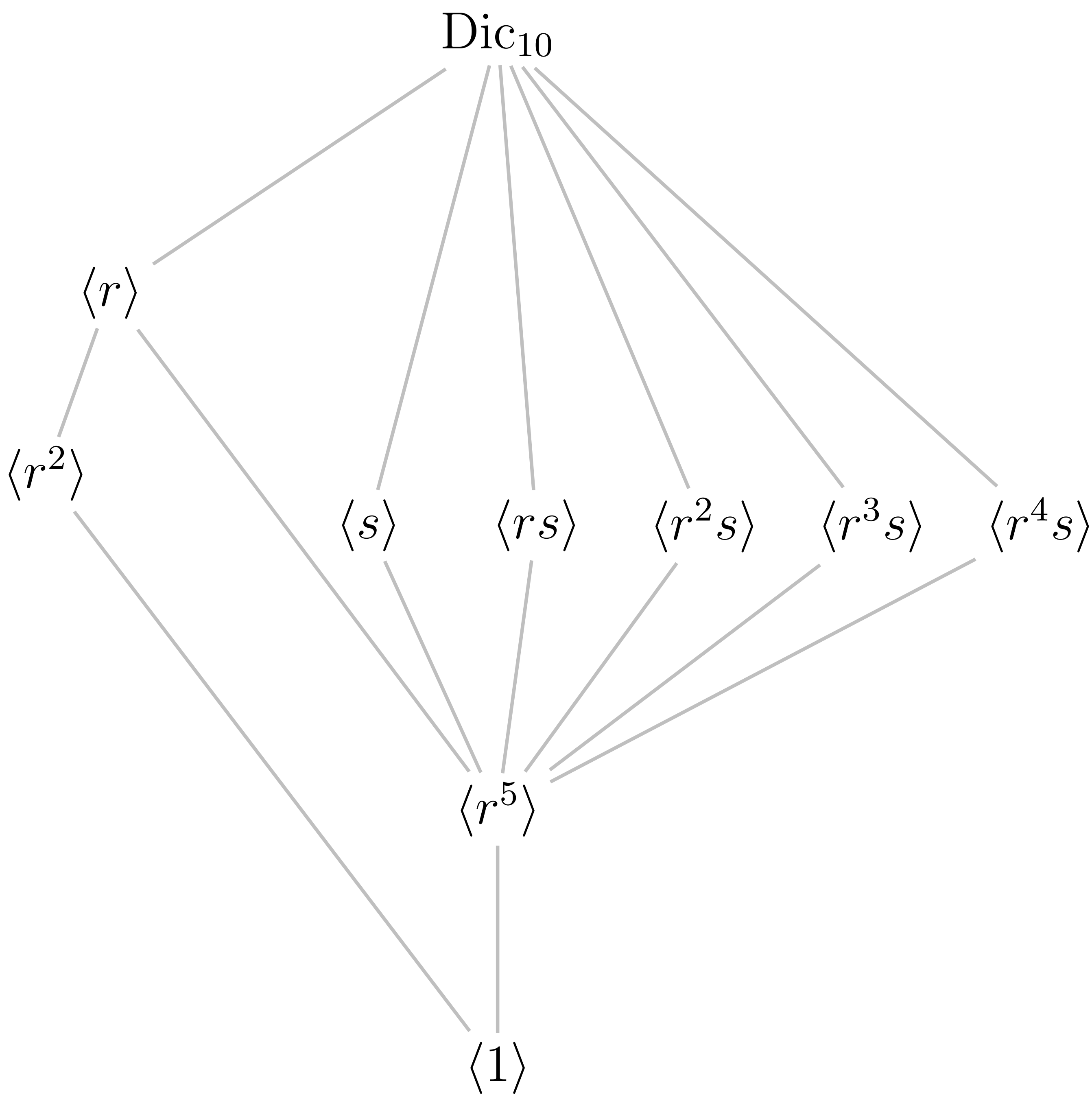
Supplemental material: Visual Algebra (Math 4120), HW 9

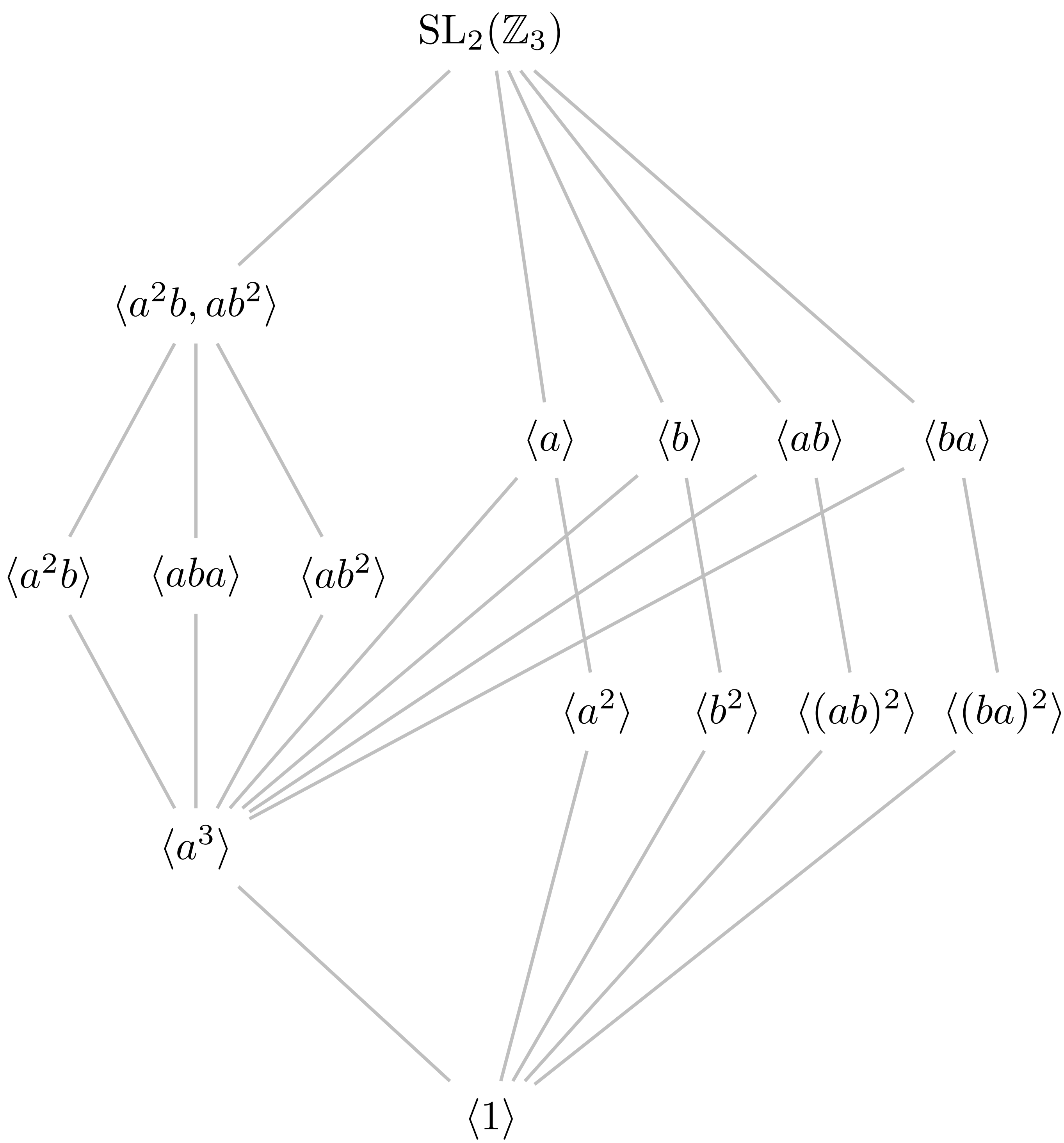
#2(c): Subgroup lattice of the *affine general linear group* $G = \mathrm{AGL}_1(\mathbb{Z}_5)$, grouped by conjugacy classes, with the k^{th} commutator subgroups $G^{(k)}$ included, and sublattice of the quotients $G^{(k)}/G^{(k-1)}$ identified, along with its isomorphism type.



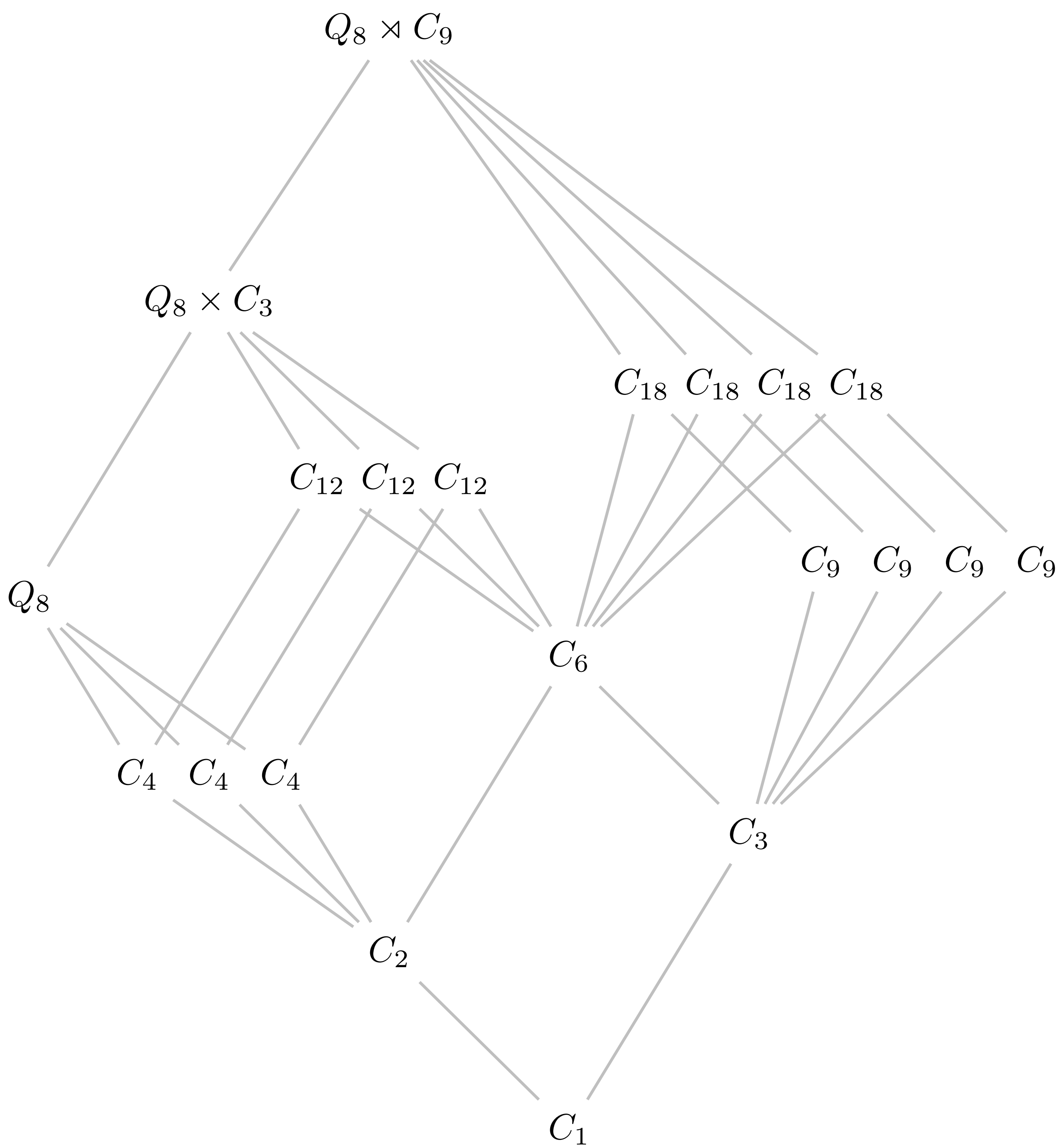
#2(c): Subgroup lattice of the *dicyclic group* Dic_{10}, grouped by conjugacy classes, with the k^{th} commutator subgroups $G^{(k)}$ included, and sublattice of the quotients $G^{(k)}/G^{(k-1)}$ identified, along with its isomorphism type.



#2(c): Subgroup lattice of the *special linear group* $G = \mathrm{SL}_2(\mathbb{Z}_3)$, grouped by conjugacy classes, with the k^{th} commutator subgroups $G^{(k)}$ included, and sublattice of the quotients $G^{(k)}/G^{(k-1)}$ identified, along with its isomorphism type.

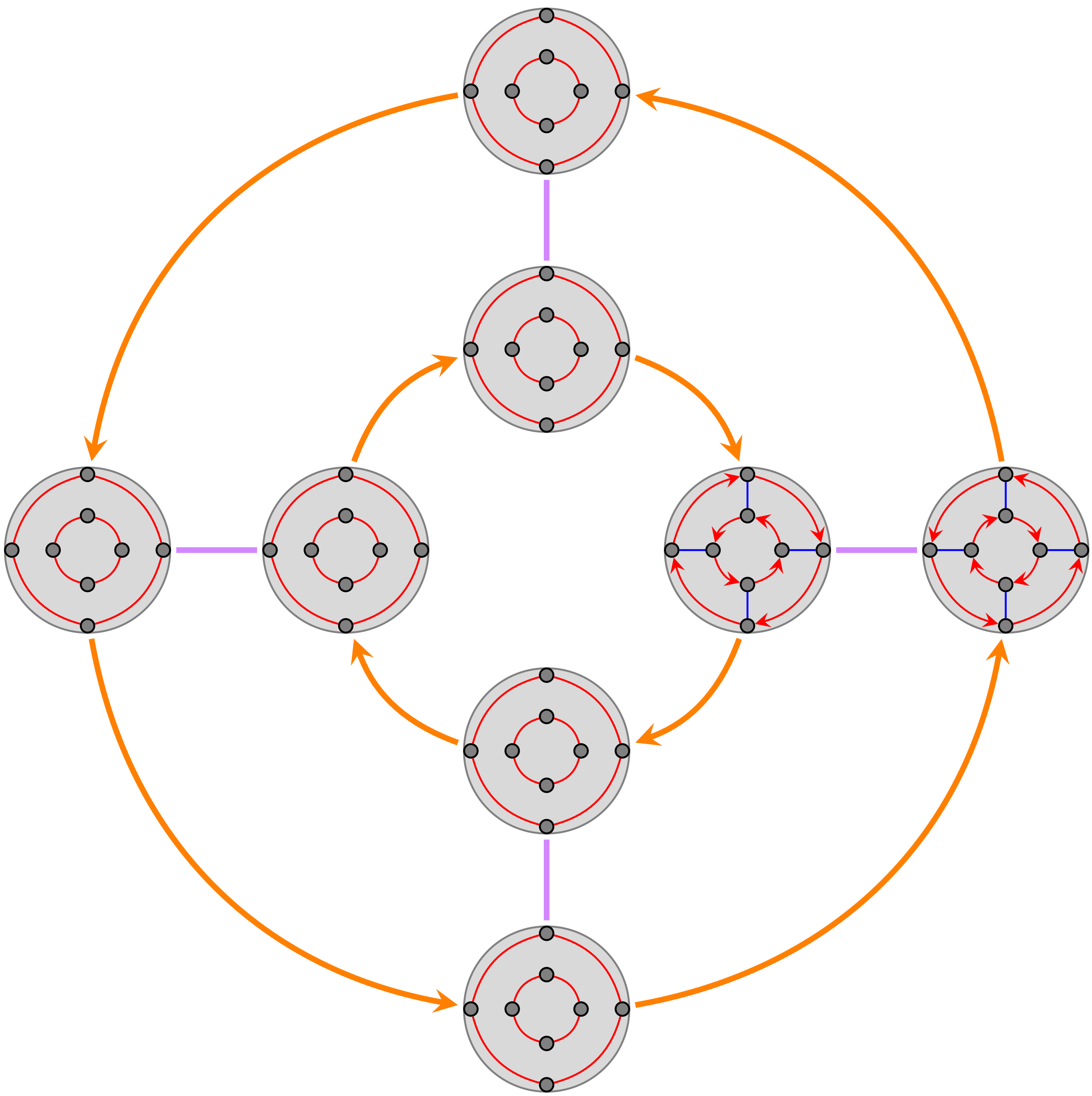


#2(c): Subgroup lattice of $G = Q_8 \rtimes C_9$, grouped by conjugacy classes, with the k^{th} commutator subgroups $G^{(k)}$ included, and sublattice of the quotients $G^{(k)}/G^{(k-1)}$ identified, along with its isomorphism type.



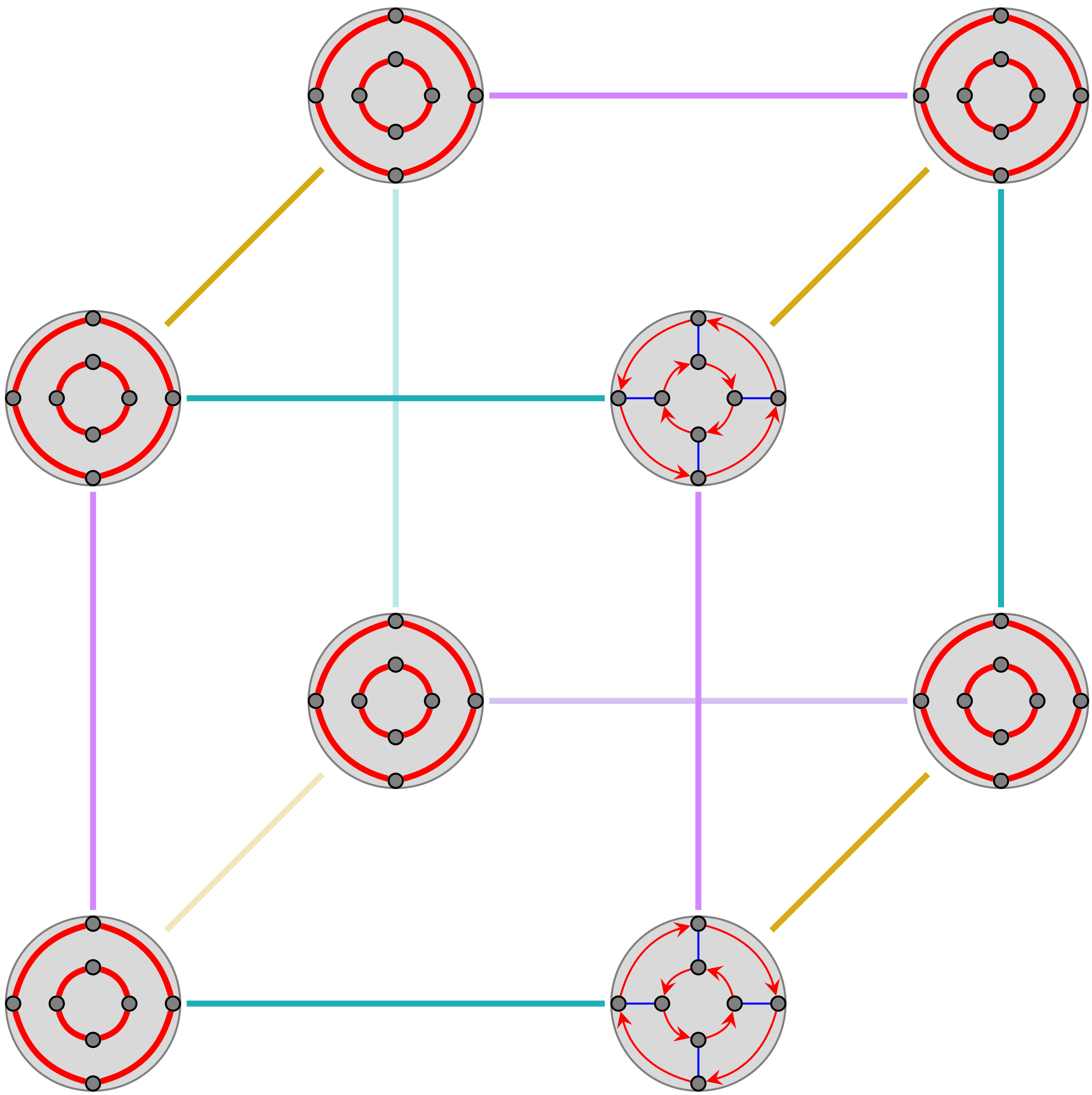
#3(a,c): Cayley graph of the automorphism group $\mathrm{Aut}(D_4) \cong D_4$, with the nodes labeled by re-wired copies of the Cayley graph of $D_4 = \langle r, f \rangle$, and also denoted with the corresponding element from

$$\mathrm{Aut}(D_4) = \left\{ \mathrm{Id},\, \varphi_r,\, \varphi_f,\, \varphi_{rf},\, \omega,\, \varphi_r\omega,\, \varphi_f\omega,\, \varphi_{rf}\omega \right\} = \mathrm{Inn}(D_4) \cup \mathrm{Inn}(D_4)\omega.$$



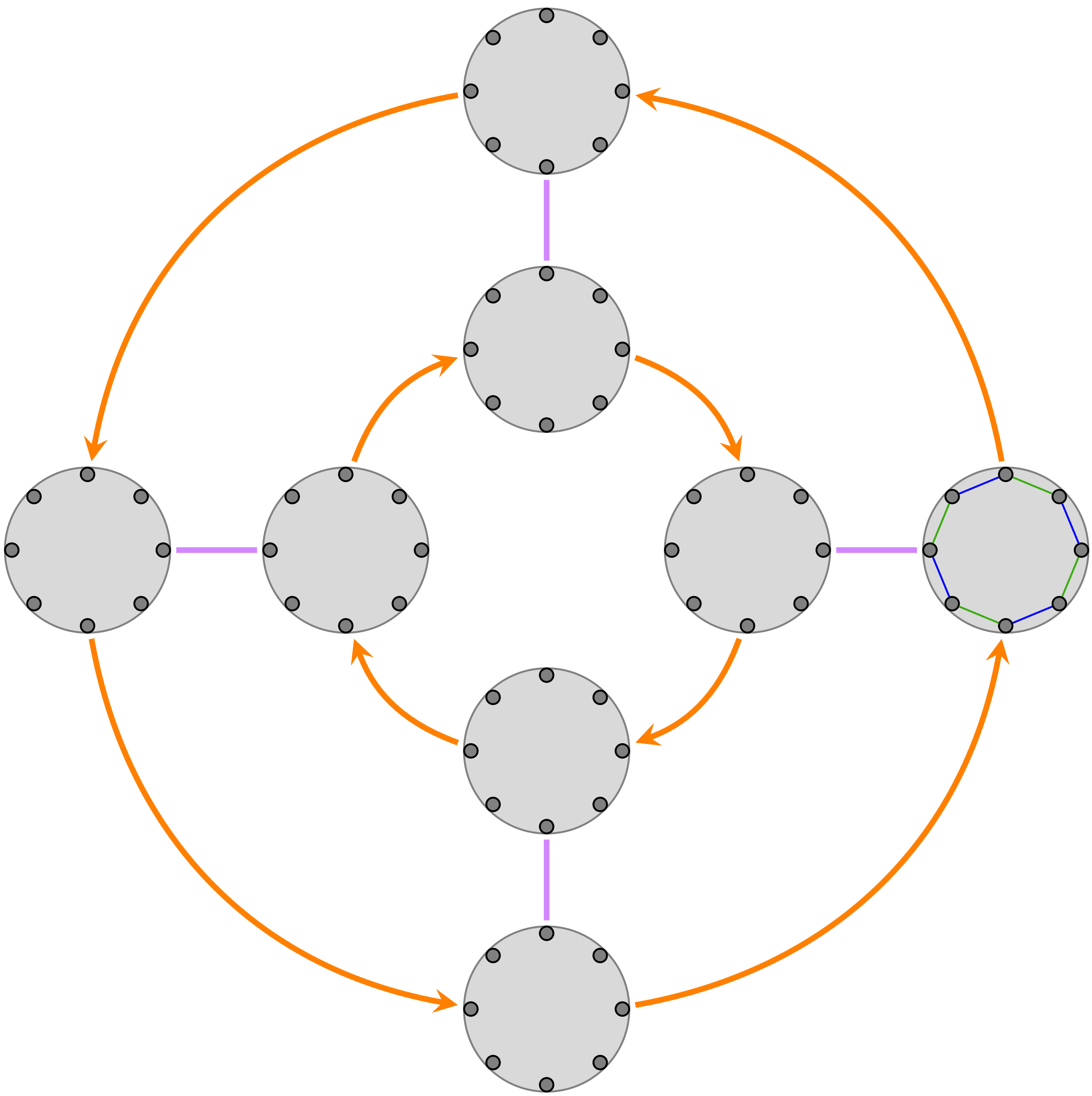
#3(a,c): Cayley graph of the automorphism group $\mathrm{Aut}(D_4) \cong V_4 \rtimes C_2 \cong D_4$, with the nodes labeled by re-wired copies of the Cayley graph of $D_4 = \langle r, f \rangle$, and also denoted with the corresponding element from

$$\mathrm{Aut}(D_4) = \left\{ \mathrm{Id},\, \varphi_r,\, \varphi_f,\, \varphi_{rf},\, \omega,\, \varphi_r\omega,\, \varphi_f\omega,\, \varphi_{rf}\omega \right\} = \mathrm{Inn}(D_4) \cup \mathrm{Inn}(D_4)\omega.$$



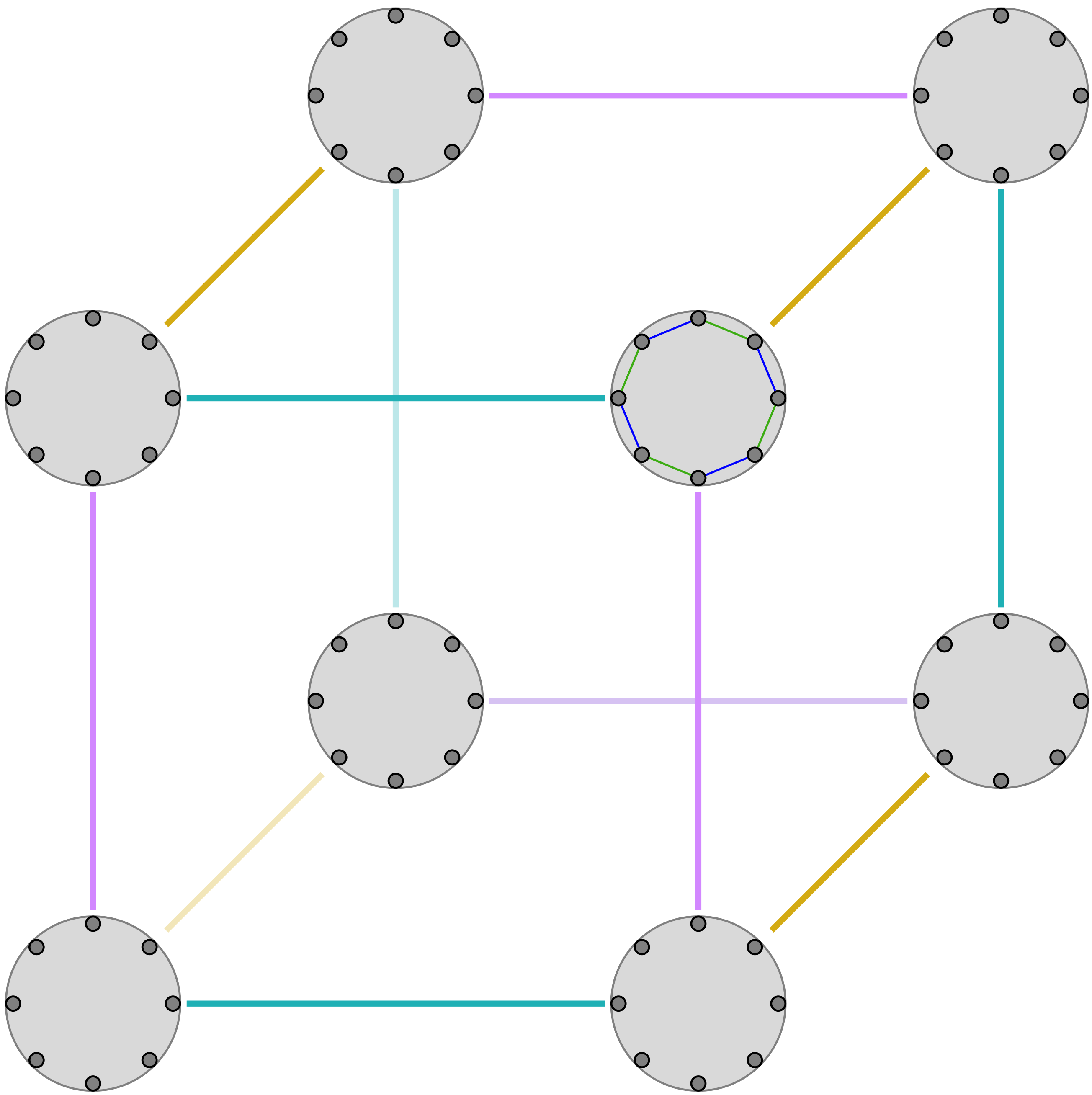
#3(b,c): Cayley graph of the automorphism group $\mathrm{Aut}(D_4) \cong V_4 \rtimes C_2 \cong D_4$, with the nodes labeled by re-wired copies of the Cayley graph of $D_4 = \langle {\color{red}r}, {\color{blue}f} \rangle$, and also denoted with the corresponding element from

$$\mathrm{Aut}(D_4) = \left\{ \mathrm{Id}, \varphi_r, \varphi_f, \varphi_{rf}, \omega, \varphi_r\omega, \varphi_f\omega, \varphi_{rf}\omega \right\} = \mathrm{Inn}(D_4) \cup \mathrm{Inn}(D_4)\omega.$$



#3(b,c): Cayley graph of the automorphism group $\mathrm{Aut}(D_4) \cong V_4 \rtimes C_2 \cong D_4$, with the nodes labeled by re-wired copies of the Cayley graph of $D_4 = \langle {\color{blue}s}, {\color{green}t} \rangle$, and also denoted with the corresponding element from

$$\mathrm{Aut}(D_4) = \left\{ \mathrm{Id},\, \varphi_r,\, \varphi_f,\, \varphi_{rf},\, \omega,\, \varphi_r\omega,\, \varphi_f\omega,\, \varphi_{rf}\omega \right\} = \mathrm{Inn}(D_4) \cup \mathrm{Inn}(D_4)\omega.$$



#4(i–ii): Both $D_3 \times C_2$ and $D_3 \times C_2$ are semidirect products, and each is defined by a "labeling map"

$$\theta\colon C_2 \longrightarrow \mathrm{Aut}(D_3) = \langle \alpha, \beta \mid \alpha^3 = \beta^2 = (\alpha\beta)^2 = 1 \rangle \cong D_3.$$

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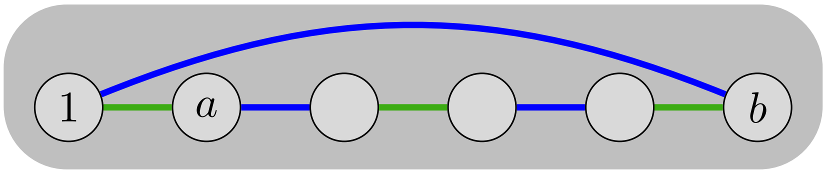
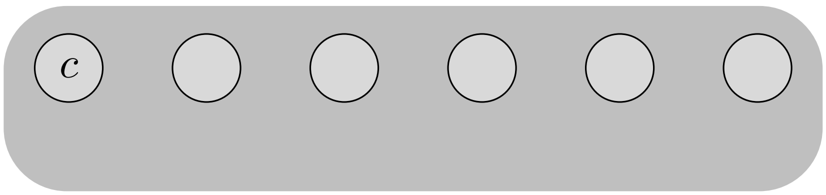
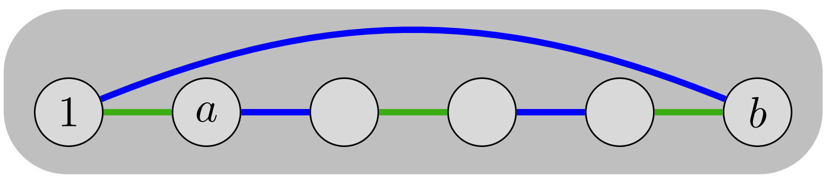


$\theta(1) = 1$

$\mathbf{D_3 \times C_2}$

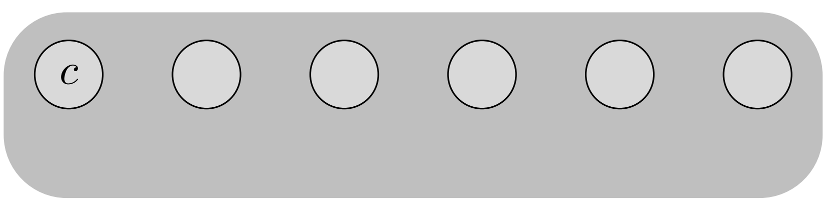


$\theta(c) =$



$\theta(1) = 1$

$\mathbf{D_3 \rtimes C_2}$



$\theta(c) =$

#4(iii–iv): The semidirect product $A \rtimes B$ is defined by "labeling map" $\theta\colon B \longrightarrow \mathrm{Aut}(A)$. Here are $V_4 \rtimes C_3$ and $C_3 \rtimes V_4$ and $\mathrm{Aut}(V_4) = \langle \alpha, \beta \rangle \cong D_3$ and $\mathrm{Aut}(C_3) = \langle 1, \phi \rangle \cong C_2$.