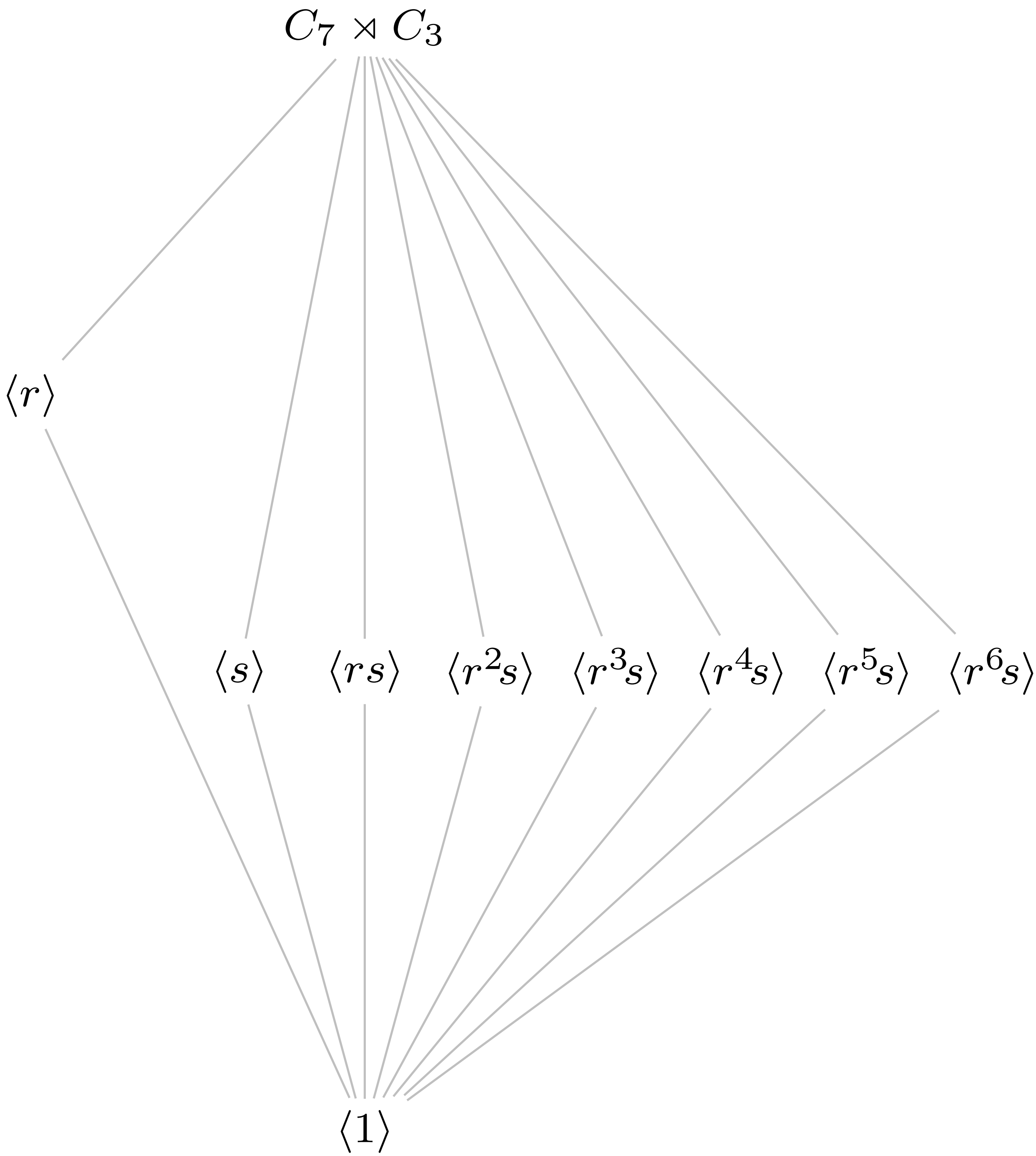
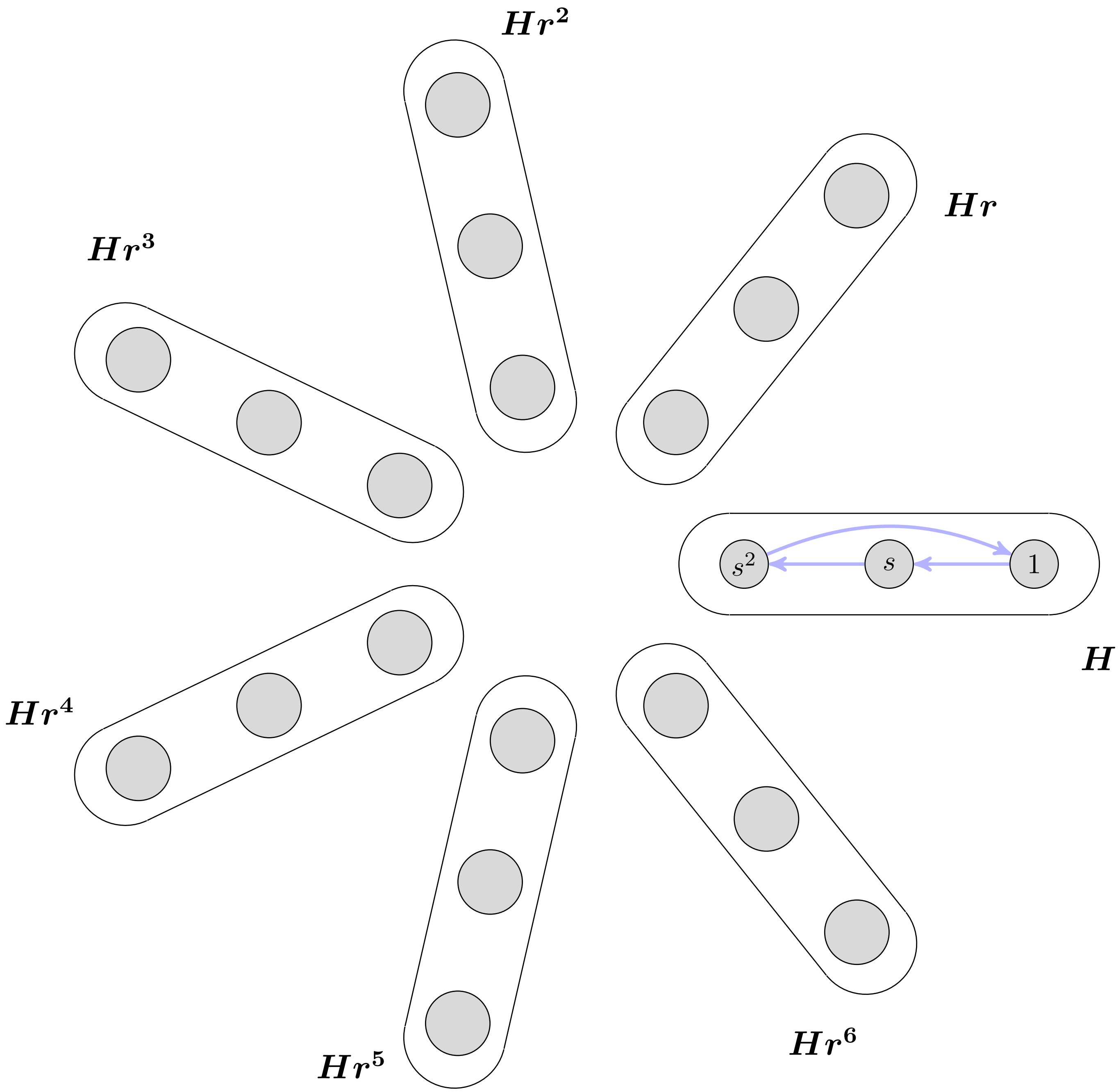
Supplemental material: Visual Algebra (Math 4120), HW 11

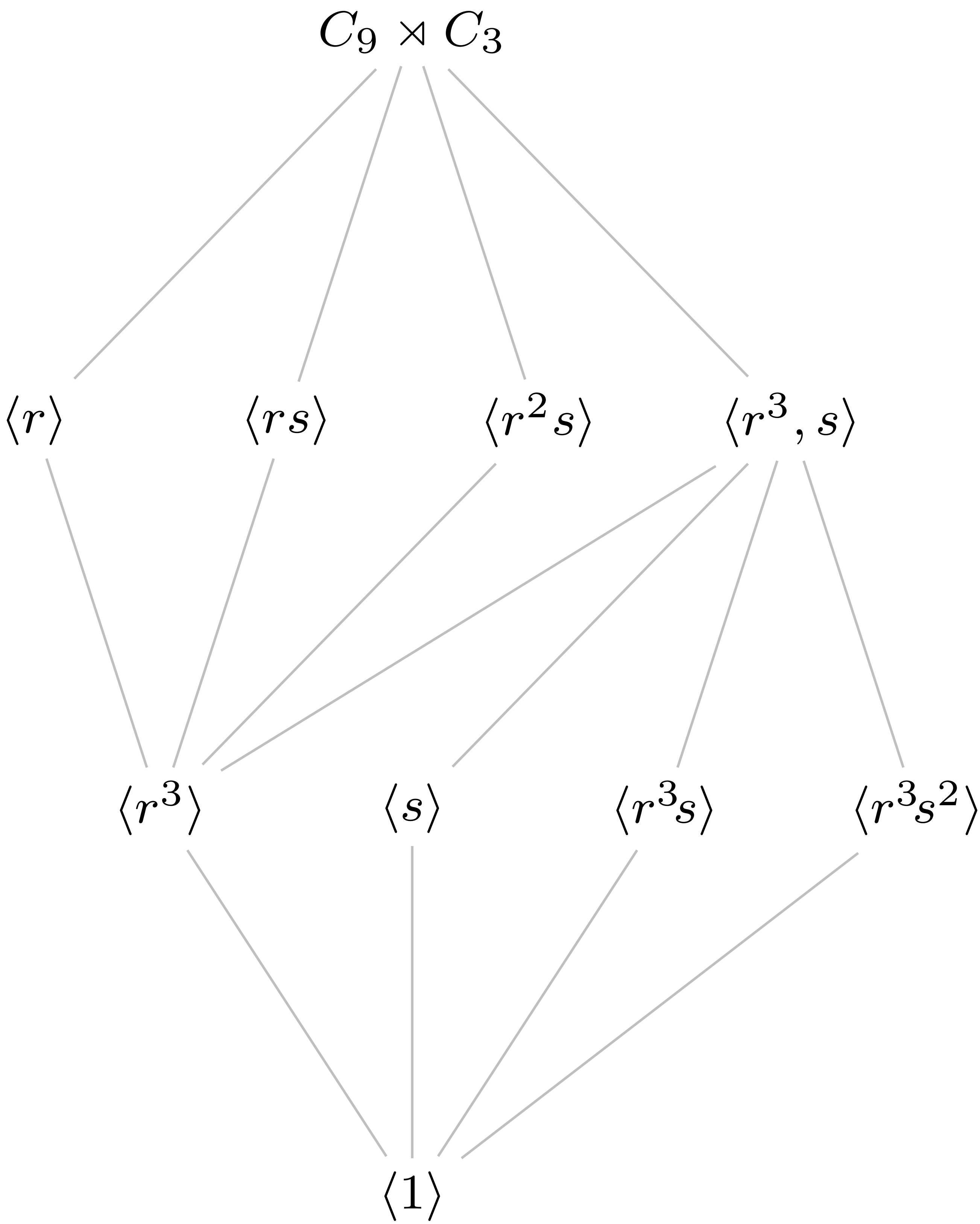
#1(a): Action graph of $C_7 \rtimes C_3$ acting on its subgroups by conjugation.



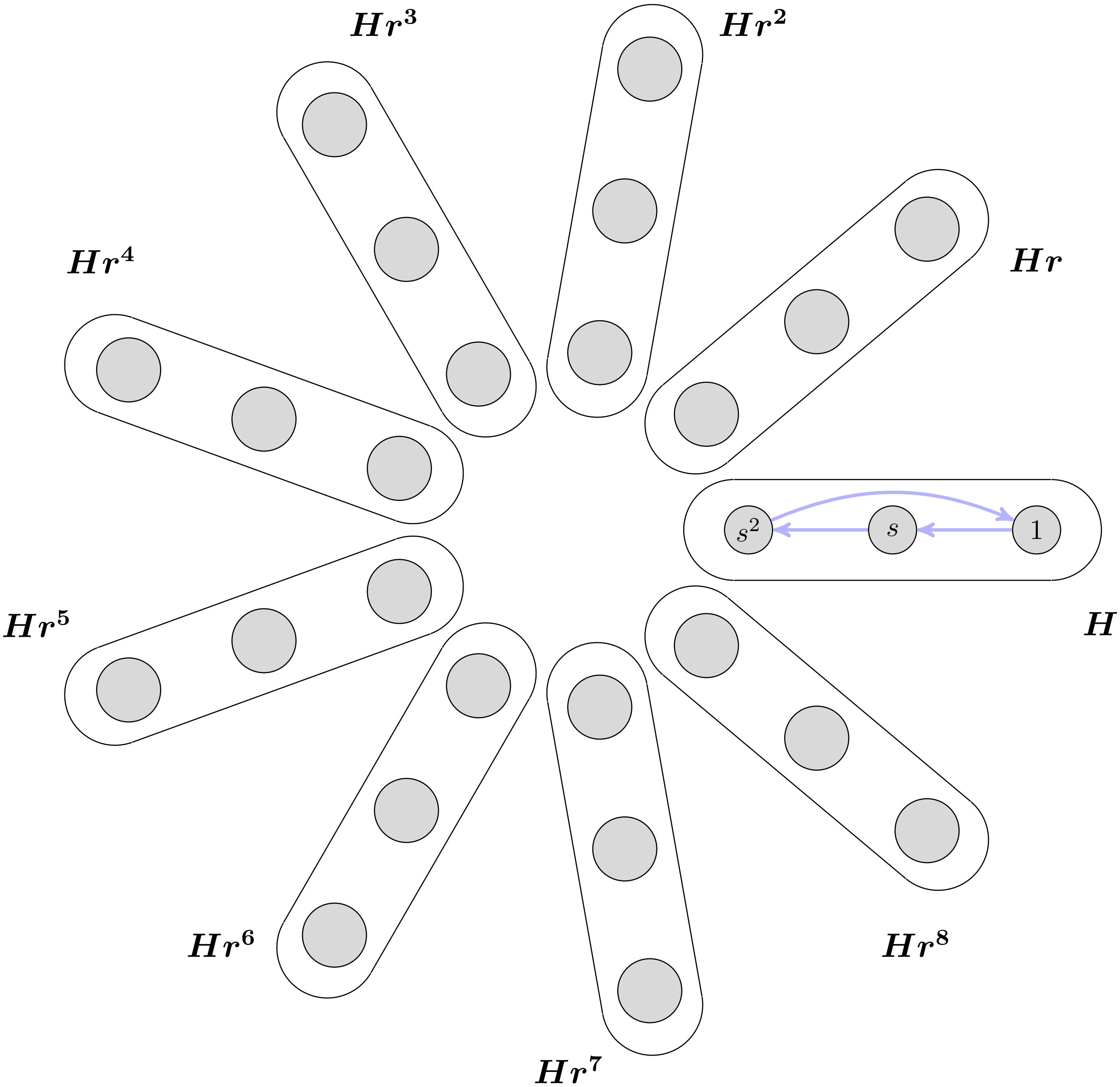
#1(b): Action graph of $C_7 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.



#**1(a)**: Action graph of $C_9 \rtimes C_3$ acting on its subgroups by conjugation.



#1(b): Action graph of $C_9 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.



#3(a): Action graph and fixed point table of $\text{Dic}_6 = \langle r, s \rangle$ acting on itself by conjugation.

1	r	r^2	s	r^2s	r^4s
r^3	r^5	r^4	rs	r^3s	r^5s

	1	r	r^2	r^3	r^4	r^5	s	rs	r^2s	r^3s	r^4s	r^5s
1												
r												
r^2												
r^3												
r^4												
r^5												
s												
rs												
r^2s												
r^3s												
r^4s												
r^5s												

#3(b): Partition of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ into cosets of $\mathrm{Inn}(\mathrm{Dic}_6)$.

	$\mathrm{Inn}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s \rangle$						$\mathrm{Inn}(\mathrm{Dic}_6)\omega$						
Id	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	ω
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	
φ_r	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	$\varphi_r\omega$
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	
φ_{r^2}	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	$\varphi_{r^2}\omega$
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	
φ_s	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	$\varphi_f\omega$
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	
φ_{rs}	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	$\varphi_{rs}\omega$
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	
φ_{r^2s}	1	r	r^2	s	r^2s	r^4s	1	r	r^2	s	r^2s	r^4s	$\varphi_{r^2s}\omega$
	r^3	r^5	r^4	rs	r^3s	r^5s	r^3	r^5	r^4	rs	r^3s	r^5s	

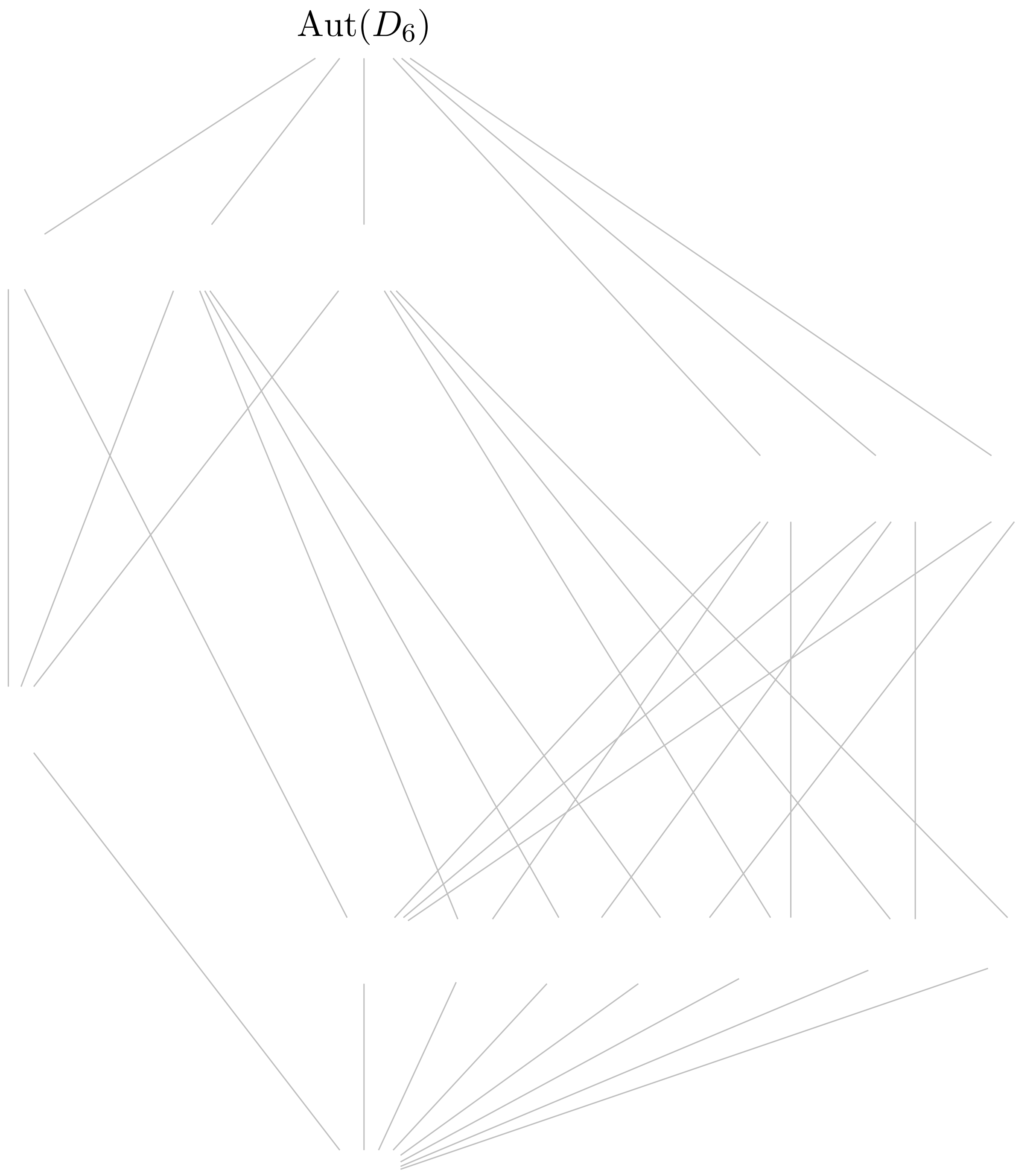
#3(b): Action graph and fixed point table of $G = \mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ acting on $S = \mathrm{Dic}_6$, where ω is the outer automorphism defined by

$$\omega\colon\ \mathrm{Dic}_6 \longrightarrow \mathrm{Dic}_6, \qquad \omega(r) = r, \quad \omega(s) = s^{-1} = r^3 s.$$

1	r	r^2	s	$r^2 s$	$r^4 s$
r^3	r^5	r^4	rs	$r^3 s$	$r^5 s$

	1	r	r^2	r^3	r^4	r^5	s	rs	$r^2 s$	$r^3 s$	$r^4 s$	$r^5 s$
Id												
φ_r												
φ_{r^2}												
φ_s												
φ_{rs}												
$\varphi_{r^2 s}$												
ω												
$\varphi_r \omega$												
$\varphi_{r^2} \omega$												
$\varphi_s \omega$												
$\varphi_{rs} \omega$												
$\varphi_{r^2 s} \omega$												

#3(c): Cayley graph and subgroup lattice of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_f, \omega \rangle \cong D_6$.



#3(d): Action graph and fixed point table of the action of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ on the conjugacy classes of Dic_6.

	$\mathrm{cl}(1)$	$\mathrm{cl}(r^3)$	$\mathrm{cl}(r)$	$\mathrm{cl}(r^2)$	$\mathrm{cl}(s)$	$\mathrm{cl}(rs)$
Id						
φ_r						
φ_{r^2}						
φ_s						
φ_{rs}						
φ_{r^2s}						
ω						
$\varphi_r\omega$						
$\varphi_{r^2}\omega$						
$\varphi_s\omega$						
$\varphi_{rs}\omega$						
$\varphi_{r^2s}\omega$						