1. Consider the following instance of the discrete logistic equation:

$$P_{t+1} = P_t(1 + r(1 - P_t/M))$$

Find the two equilibirum points, $P^*$. Use the technique of *linearization* to find the stability of these points. That is, plug $P_t \approx P^* + p_t$ and $P_{t+1} \approx P^* + p_{t+1}$ into the difference equation and express the perturbation $p_{t+1}$ in terms of $p_t$, disregarding the nonlinear terms.

2. In this problem, we will investigate 2-cycles in the logistic map $f(x) = rx(1 - x)$.

   (a) The presence of a 2-cycle $p, q, p, q, \ldots$ means that $p$ is a fixed point of $f^2(x) := f(f(x))$, and hence a root of the polynomial $f^2(x) - x = 0$. Find this polynomial.

   (b) The fixed points of the logistic map, $x^* = 0$ and $x^* = \frac{r-1}{r}$, are both roots of this polynomial. Therefore,

$$f^2(x) = x\big(x - \tfrac{r-1}{r}\big)g(x).$$

   Find $g(x)$ (feel free to use a computer) and its roots.

   (c) Determine for what values of $r$ give rise to a 2-cycle, with justification.

   (d) The 2-cycle $p, q, p, q, \ldots$ of $f$ is *stable* if $p$ and $q$ are stable fixed points for $f^2$. Compute

$$\lambda := \frac{d}{dx}(f^2(x))_{x=p}.$$

   Then substibute $r$ back in, and determine when $|\lambda| < 1$. This gives the values of $r$ for which the 2-cycle is stable.

3. A population model has an *Allee effect* is if $F(P_t) < P_t$ for small values of $P_t$. One example of this is the following:

$$\Delta P/P = rP(M - P)(P - T)$$

   (a) Sketch a graph of $P_{t+1}$ vs. $P_t$, and of $\Delta P/P$ vs. $P$ (*per capita growth rate*), and compare these to the logistic model.

   (b) Describe what features of a population this model is capturing?

   (c) Investigate the resulting model using the Mathlab programs `onepop` and `cobweb` (available on the Math 4500 webpage) for some choices of $r$, $M$, and $T$. Can you find any values that lead to interesting dynamics (doubling, chaos, etc.)? Print out (accurately!) sketch a few examples.

   (d) What features of this modeling equation are unrealistic? How might the model be improved?

4. Repeat the previous problem but for the *Ricker equation*

$$\Delta P = P(e^{r(1-P/M)} - 1)$$