1. Imagine a predator–prey interaction in which a certain number of the prey population cannot be eaten because of a refuge in their environment that the predator cannot enter. Give several real-life examples of two populations that might exhibit this feature, and propose a model of this. Either an ODE of difference equation framework is fine.

2. Consider the following model of two species:

$$\frac{dX}{dt} = r_1 X\left(1 - \frac{X}{M_1 + b_{12}Y}\right)$$
$$\frac{dY}{dt} = r_1 Y\left(1 - \frac{Y}{M_2 + b_{21}X}\right).$$

   (a) Describe what interactive behavior between species X and Y is implied by the model, and an example of two species that this might model. Include a discussion of what the parameters might represent.

   (b) Find the nullclines, $X' = 0$ and $Y' = 0$, and sketch them on the XY-plane.

   (c) Find the steady states of this model, and determine their stability by linearization.

   (d) Summarize the ecological implications of your results.

3. Consider the system of difference equations $\begin{cases} P_{t+1} = P_t(1 + 1.3(1 - P_t)) - .5P_tQ_t \\ Q_{t+1} = .3Q_t + 1.6P_tQ_t \end{cases}$

   (a) Describe what type of interacting species this models, and why.

   (b) Find the steady-states, and linearize them to determine their stability.

   (c) Plot the solutions using the MATLAB program `twopop`, which is available on the course website.

   (d) The discrete logistic model $P_{t+1} = P_t(1 + 1.3(1 - P_t))$ has underdamped dynamics (=damped oscillations) when $r = 1.3$, but overdamped dynamics when $r < 1$. Using the `twopop` program, can you find a value of r that yields no oscillations in the interacting species model above? If so, what is the approximate threshold of r between these two dynamical regimes? Include printouts, screenshots, or accurate sketches to support your hypothesis.