1. Suppose a disease is modeled by the SIR model, but many people who get the disease self-quarantine. The net result is that a fraction $q \geq 0$ of the infectives are prevented from having contacts with the susceptibles.

   (a) Modify the equations of the standard SIR model to reflect this.

   (b) Quarantining can be viewed as a way of modifying the transmission coefficient. Suppose an SIR model has transmission coefficient β, and a fraction q of the infectives are successfully quarantined. Then the model with quarantining is identical to a standard SIR model with some other transmission coefficient β', the *effective transmission coefficient*. Give a formula for β' in terms of β and q.

   (c) Repeat the previous parts using the SIR model with births and deaths. Use μ for the constant birth rate and uniform death rate.

   (d) Use MATLAB (e.g., Program 2.1 & 2.2) of Keeling/Rohani) to investigate the behavior of your quarantine models for some fixed β, and γ, and vary q from 0 to 1. Explain the qualitative behavior you see.

   (e) Pick two common diseases that might be well-suited for the SIR model, with quite different values of $\mathcal{R}_0$. For each one, experiment with various values of q (at least 5) and report back how much of a difference this makes in the final epidemic size (classic model), and in the endemic equilibrium (model with births and deaths).

   (f) In the previous part, can you find a value of q that prevents an epidemic from occurring, regardless of I_0? Does this depend on whether there are births and deaths? Justify your answer mathematically.

2. Since vampires have circulated endemically in the human population for thousands of years, they are well-suited for the following *SVA* model:

$$\frac{dS}{dt} = \mu - \beta SV - \delta_S S$$
$$\frac{dV}{dt} = \beta SV - \delta_V V$$
$$\frac{dA}{dt} = \alpha\delta_V V - \delta_A A,$$

   where $\alpha \in [0,1]$. The "A" class represents the *abstinent* vampires that are not sucking blood, presumably because they are no longer willing or able to hunt. Of course, this will eventually lead to their demise.

   (a) Carefully justify this model. Make sure you explain what the rate constants represent, and why the terms are what they are. Include a diagram showing each compartment and the transitions.

   (b) Find the basic reproductive number $\mathcal{R}_0$, for which a vampire epidemic will spread if $\mathcal{R}_0 > 1$ but die out if $\mathcal{R}_0 < 1$.

   (c) What other disease(s) might be well-modeled by these equations? Fully justify your answer.

3. Whereas vampires circulate among humans almost unnoticed in an endemic manner, other members of the living dead, such as zombies, occasionally pop up in brief but virulent outbreaks. The possibility of deceased individuals resurrecting back into zombies requires a different approach, such as the following SZR model:

$$\frac{dS}{dt} = \mu - \beta SZ - \delta S$$
$$\frac{dZ}{dt} = \beta SZ + \zeta R - \alpha SZ$$
$$\frac{dR}{dt} = \delta S + \alpha SZ - \zeta R$$

   (a) Carefully justify this model. Make sure you explain what the rate constants represent, and why the terms are what they are. Include a diagram showing each compartment and the transitions.

   (b) Compute the Jacobian at the zombie-free equilibrium $(S, Z, R) = (1, 0, 0)$ and determine its stability.

   (c) Compute the Jacobian at the doomsday equilibrium $(S, Z, R) = (0, Z^*, 0)$ and determine its stability.

   (d) Discuss the implications of the previous parts of this problem on humanity. Is there any way to save us?