Problem 1. Suppose a, b, and c are integers. Prove that if $a \mid b$ and $a \mid c$, then $a \mid (b+c)$.

Problem 2. Prove that the sum of two odd integers is even.

Problem 3. Consider the statement:

If $x, y \in \mathbb{Z}$ are odd, then xy is odd.

Prove this three different ways: via a direct proof, a proof by contradiction, and by proving the contrapositive.

Problem 4. Consider the following "theorem," and subsequent "proof."

Theorem. The money you have is equal to the money that you need.

Proof. Let M be the money that you have and N the money that you need. We will show that $M = N$. Consider the average of these two quantities, $A = \frac{N+M}{2}$. We have

$$2A = M + N,$$

and multiplying both sides of the equation above by $M - N$ yields

$$2AM - 2AN = M^2 - N^2.$$

Equivalently, we can write $N^2 - 2AN = M^2 - 2AM$. Adding A^2 to both sides gives

$$N^2 - 2AN + A^2 = M^2 - 2AM + A^2,$$

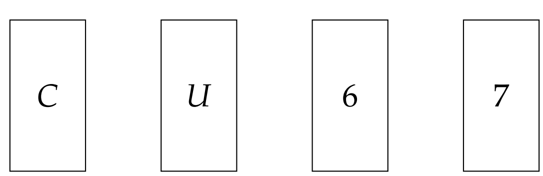
or equivalently,

$$(N - A)^2 = (M - A)^2.$$

Taking the square root and adding A to both sides gives $N = M$. □

Pinpoint the flaw in this "proof" as precisely as you can.

Problem 5. Consider four cards, each of which has a letter on one side and a number on the other side. They are placed on the table in front of you, and you can see the following:



What is the fewest number of cards that you need to flip over to verify the statement:

"*if a card has a vowel on one side, then it has an even number on the other side*"?

Fully justify your answer.