

**Problem 1.** Prove that for every pair of positive integers  $a$  and  $b$ , their greatest common divisor is unique.

**Problem 2.** The  $n \times n$  zero matrix  $\mathbf{0}$  has the property that  $\mathbf{A} + \mathbf{0} = \mathbf{A}$  for all  $n \times n$  matrices  $\mathbf{A}$ . Prove that this is the unique  $n \times n$  matrix with this property.

**Problem 3.** Prove that  $n^2$  is odd if and only if  $n^2 - 1$  is divisible by 8.

**Problem 4.** Prove that  $x < y$  if and only if there exists  $\epsilon > 0$  such that  $x + \epsilon < y$ . Then, explain why this implies that  $0.999\dots = 1$ .