Problem 1. Prove that for all integers $n \geq 1$,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 2. Prove the following inequality for all integers $n \geq 2$:

$$\sqrt{n} < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}}.$$

Problem 3. Recall that the Fibonacci numbers are defined by the recurrence $F_{n+2} = F_{n+1} + F_n$, with $F_1 = 1$ and $F_2 = 1$. Prove that

$$F_n \leq 2^{n-1} \qquad \text{for all } n \geq 1.$$

Problem 4. Prove that for any $n+1$ distinct numbers, there will be two whose difference is divisible by n.

Problem 5. Construct truth tables that demonstrate the following tautologies, and include the intermediate steps as columns.

(i) *Modus tollens*: $\big[\neg Q \;\wedge\; (P \Rightarrow Q)\big] \;\Rightarrow\; \neg P$

(ii) Contradiction: $P \Leftrightarrow [(\neg P) \Rightarrow (Q \wedge \neg Q)]$

Rather than include this as an exercise in your manuscript, weave it into your narrative directly in the chapter, right after *modus ponens*.