Problem 1. Rewrite each of the following sets by listing their elements between braces. If it is an infinite set, write out enough elements for the reader to see the pattern, then use ellipses.

(a) $\{5x+2 : x \in \mathbb{Z}\}$

(b) $\{x \in \mathbb{Z} : -2 \leq x < 7 \dots\}$

(c) $\{5a+2b : a, b \in \mathbb{Z}\}$

(d) $\left\{\frac{m}{n} \in \mathbb{Q} : \left|\frac{m}{n}\right| < 1 \text{ and } 1 \leq n \leq 4\right\}$

(e) $\left\{x \in \mathbb{R} \mid x^4 = 1\right\}$

(f) $\left\{x \in \mathbb{C} \mid x^4 = 1\right\}$

(g) $\{3n : n \in \mathbb{Z} \text{ and } |2n| < 8\}$

(h) $\{x \in \mathbb{R} : \sin(\pi x) = 0\}$

Problem 2. Rewrite each of the following sets in set-builder notation.

(a) $\{3, 5, 7, 9, 11, \dots\}$

(b) $\{-2, -1, 0, 1, 2, 3, 4, 5\}$

(c) $\{\dots, -\frac{3\pi}{2}, \pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{3\pi}{2}, \dots\}$

(d) $\{\dots, -\frac{8}{27}, -\frac{4}{9}, -\frac{2}{3}, 1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots\}$

(e) $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\}$

(f) $\{0, 4, 16, 36, 64, 100, \dots\}$

(g) $\{2, 4, 8, 16, 32, 64, \dots\}$

(h) $\{2, 11, 101, 1001, 10001, \dots\}$

Problem 3. For each for the following sets, use TikZ to sketch the points/arcs/regions in the xy-plane.

(a) $\left\{(x, y) : x \in [1, 3],\ y \in [-1, 2]\right\}$

(b) $\left\{(x, y) : x \in \mathbb{Z},\ y \in \mathbb{R}\right\}$

(c) $\left\{(x, y) : x^2 + y^2 = 4\right\}$

(d) $\left\{(x, y) : x^2 + y^2 \leq 4\right\}$

(e) $\{1, 2, 3\} \times \{1, 4\}$

(f) $\{1, 2, 3\} \times [1, 4]$.

Problem 4. Use elementary logic to show that $\neg(A \subsetneq B)$ is equivalent to

$$(A \not\subseteq B) \vee (A = B).$$

Problem 5. Using elementary logic, and the equivalence of $A = B$ and $(A \subseteq B) \wedge (B \subseteq A)$, show that $A \neq B$ is equivalent to

$$(\exists a)\big[\big((a \in A) \wedge (a \notin B)\big) \vee \big((a \in B) \wedge (a \notin A)\big)\big].$$