Problem 1. For each $r > 0$, define the *open interval of radius* r as the set

$$I_r = (-r, r) \subseteq \mathbb{R}.$$

Determine the following unions and intersections. Briefly justify each answer.

(a) $\displaystyle\bigcup_{r>0} I_r$ (b) $\displaystyle\bigcap_{r>0} I_r$ (c) $\displaystyle\bigcup_{0<r<1} I_r$ (d) $\displaystyle\bigcap_{r>1} I_r$

Then, repeat these, but for the *closed interval of radius* r, defined as $\overline{I_r} = [-r, r]$.

Problem 2. For each $r > 0$, define the open and closed disks of radius r as the following subsets of $\mathbb{R}^2$:

$$D_r = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < r^2\}, \qquad \overline{D_r} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq r^2\}.$$

Determine the following unions and intersections. Briefly justify each answer.

(a) $\displaystyle\bigcup_{r>0} D_r$ (b) $\displaystyle\bigcap_{r>0} D_r$ (c) $\displaystyle\bigcup_{0<r<1} D_r$ (d) $\displaystyle\bigcap_{r>1} D_r$

Then, repeat them for the closed disks. Many of your answers will be in terms of open or closed disks.

Problem 3. Prove that

$$\{n \in \mathbb{Z} : 2 \mid n\} \cap \{n \in \mathbb{Z} : 9 \mid n\} \subseteq \{n \in \mathbb{Z} : 6 \mid n\}.$$

Problem 4. Suppose $I \neq \emptyset$ and $I \subseteq J$. Prove the following containments:

(a) $\displaystyle\bigcup_{\alpha\in I} A_\alpha \subseteq \bigcup_{\alpha\in J} A_\alpha$ (b) $\displaystyle\bigcap_{\alpha\in I} A_\alpha \supseteq \bigcap_{\alpha\in J} A_\alpha$.

Problem 5. Let A and B be sets.

(a) Show that $\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$.

(b) State and prove a general condition on A and B for which equality holds in Part (a).