

Problem 1. Let $X = \mathbb{Z}$ and let \equiv be the relation where $a \equiv b$ iff $n \mid (a - b)$. The set of equivalence classes

$$\{[0], [1], \dots, [n-1]\}$$

forms a ring, where addition and multiplication are defined as

$$[a] + [b] = [a + b], \quad \text{and} \quad [a] \cdot [b] = [a \cdot b].$$

Prove the addition is *well-defined*. That is, if $[a] = [a']$ and $[b] = [b']$, then $[a + b] = [a' + b']$.

Problem 2. Let X be a set. Define an relation on the power set $P = \mathcal{P}(X)$ where

$$A \preceq B \iff A \subseteq B.$$

Prove that (P, \preceq) is a partially ordered set.

Problem 3. Prove or disprove: there exists a bijective function $f: \mathbb{Q} \rightarrow \mathbb{R}$.

Problem 4. On HW 0, you were asked: *Based on your past experience and knowledge, which AI tools do you expect to use in the class, and for which tasks?*

A lot can change in 15 weeks! How would your answer to that question be different now than it was then. Specifically, what AI tools did you end up using, and for what tasks? Were there any that you learned about since the beginning of our class and now use? For convenience, please copy/paste your response to the HW 0 question directly above your answer to this question.