MATH 3110 - Fall 2014 Homework 3

Due: Thursday September 18

ALWAYS MOTIVATE THE ANSWERS!

Questions. Chapter 3 of Strang

(total of 6 marks)

1. (a) Describe the column space of
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$
. Which subspace is it? (2 marks)

(b) Construct a
$$3 \times 3$$
 matrix whose column space contains vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and not $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (2 marks)

(c) Construct a 3×3 matrix whose column space is a line.

(2 marks)

Questions. Chapter 3 of Strang

(total of 14 marks)

1. Prove the following statements about the vector space \mathbb{M} of 3×3 matrices

(6 marks)

- (a) The set of upper triangular matrices is a subspace of M.
- (b) The union of the following sets
 - upper triangular matrices of M, and
 - lower triangular matrices of M

is *not* a vector space of \mathbb{M} . (Find an example.)

- (c) The set of symmetric matrices is a subspace of $\mathcal M$
- 2. Compute the row reduced echelon form of the following matrices

(3 marks)

$$A = \begin{pmatrix} 1 & 2 & 2 & 3 & 9 \\ 3 & 6 & 1 & 4 & 7 \\ 0 & 0 & 1 & 1 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 3 \\ 4 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

3. Find the special solutions of the nullspace of the following matrices

(3 marks)

$$A = \begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{ and } \quad B = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

4. Construct a matrix A such that N(A) contains all multiples of $\begin{pmatrix} 0\\1\\2\\1 \end{pmatrix}$. (2 marks)