MATH 3110 - Fall 2014
Homework 4

Due: Thursday September 25

Questions. Chapter 3 of Strang **(total of 20 marks)**

1. Find all 2×2 real matrices A such that $A^2 = I$. (2 marks)

2. (a) Write the 3×7 matrix in rref with the largest number of 1 as entries. (1 marks)

   (b) Write the 3×7 matrix in rref with the largest amount of 1 as entries and pivot columns 2 and 4. (1 marks)

3. Answer the following questions. (4 marks)

   (a) Find a matrix A such that the only solution of $Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

   (b) Show that it is not possible to find a matrix B such that the *only* solution of $Bx = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

4. Compute rank and set of solutions (by finding a particular solution and the nullspace) of the systems: (12 marks)

   1. $\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ 5 \end{pmatrix}$

   2. $\begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$

   3. $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 10 \end{pmatrix}$

   4. $\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$