Questions. Chapter 3 of Strang   **(total of 20 marks)**

1. Find the dimension and a basis of the following subspaces of the space of 3×3 matrices.   (6 marks)

   (a) Lower triangular matrices.

   (b) All symmetric matrices.

   (c) All anti-symmetric matrices.

2. Find a basis for the spaces $C(A)$, $N(A)$, $R(A)$ and $N(A^T)$ for   (8 marks)

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 3 & 3 & 1 & 3 & 3 \\ 1 & 1 & 3 & 3 & 1 & 3 & 3 \\ 1 & 1 & 3 & 3 & 1 & 3 & 3 \end{pmatrix}.$$

3. Let $V = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \rangle, W = \langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rangle \subseteq \mathbb{R}^3$.   (4 marks)

   Find an element of the subset $V \cap W$. Explain why $V \cap W \neq \{0\}$.

4. Without computing A, find bases for the row and column space.   (2 marks)

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{pmatrix} \begin{pmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}.$$