# MATH 3110 - Fall 2014
# Homework 7

Due: Thursday October 23

**Questions. Chapter 4.2 of Strang** **(total of 8 marks)**

1. Consider the subset $S \subseteq \mathbb{R}^4$ defined by the equation $x - y - 2z = 0$. (6 marks)

   (a) Find the dimension of $S$ and give a basis of it.

   (b) Consider the basis to be the columns of a matrix $A_1$ such that $S = C(A_1)$. Compute the projection matrix $P_1$ for $S$.

   (c) Find another basis for $S$ and compute the projection matrix $P_2$. Notice that $P_1 = P_2$, meaning that the projection matrix does not depend on the choice of the basis.

2. Show that if $P$ is a projection matrix, then $I - P$ is a projection matrix. (2 marks)

**Questions. Chapter 4.3 of Strang** **(total of 5 marks)**

1. Consider the four data points $(t_i, b_i) = (0, 0),\ (1, 8),\ (3, 8)$ and $(4, 20)$. (5 marks)
   Find the best fitting line $b = C + Dt$ between the points.

**Questions. Chapter 4.4 of Strang** **(total of 7 marks)**

1. Find a basis of the plane $2x + y - 2z = 0$ and make it orthonormal. (2 marks)

2. Using Gram-Schmidt, make the vectors $a_1 = \begin{pmatrix} 2 \\ 3 \\ 6 \\ 0 \end{pmatrix}$, $a_2 = \begin{pmatrix} 2 \\ 1 \\ 7 \\ 2 \end{pmatrix}$ and $a_3 = \begin{pmatrix} -1 \\ 3 \\ 7 \\ 4 \end{pmatrix}$ orthonormal. (3 marks)

3. Let us prove that orthonormal vectors $q_1, q_2, q_3$ are linearly independent. (2 marks)

   (a) Vector version. Using vector product, prove that if $c_1q_1 + c_2q_2 + c_3q_3 = 0$ then $c_1 = c_2 = c_3 = 0$.
   (Hint: Which vector product leads to $c_i = 0$ for $i = 1, 2, 3$?)