

MATH 3110 - Fall 2014

Homework 8

Due: Thursday October 30

Questions. Chapter 5.1 and 5.2 of Strang

(total of 20 marks)

1. Compute the determinant of the following matrices (show the computations) and in case of zero determinant (8 marks) find a linear combination of the columns of the matrix with some non zero coefficients which give the zero vector.

(a) $\begin{pmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \\ 1 & 0 & 2 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

2. Let A be a 4×4 matrix with determinant 3. Compute the determinant of the following matrices. (3 marks)

(a) $(3A)^{-1}$

(b) $((A + A)^T - A^T) * A$

3. Using only properties from Section 5.1, compute the determinant of the following matrices with respect to the parameter λ . For which values of λ are the matrices singular? (6 marks)

(a) $\begin{pmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & 3 \\ \lambda & \lambda & \lambda \end{pmatrix}$.

(b) $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} - \lambda I$ where I is the 2×2 identity matrix.

4. Show using the cofactor formula that if (3 marks)

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}, B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \text{ and } D = \begin{pmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \end{pmatrix}$$

then

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A| \cdot |D|.$$