

MATH 3110 - Fall 2015

Homework 1

Due: Thursday, September 3

Question 1. Chapter 1 of Strang

(total of 10 marks)

- Four corners of a rectangle are $(0, 0, 0)$, $(2, 0, 0)$, $(0, 6, 0)$ and $(0, 0, 4)$. (3 marks)
 - Find the remaining 4 corners.
 - Find the coordinates of the center point of the rectangle.
 - Find the center points of the six faces.
- Find two different linear combinations of the vectors $v_1 = (4, 3)$, $v_2 = (2, 1)$ and $v_3 = (2, 2)$ that produce $w = (0, 1)$. (2 marks)
- Consider the following three vectors: (3 marks)

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} \text{ and } v_3 = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$

- Find a linear combination of the vectors that give the zero vector.
 - Let V be the 3×3 with vectors v_1, v_2 and v_3 as columns. What can you say about V ?
 - Where do the three vectors lie in? (line, plane or 3d space) Motivate the answer.
 - Following Section 1.2 of Strang, compute the length of the three vectors.
- Without using elimination, find the solution of the following system of linear equations. (2 marks)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

Question 2. Chapter 2 of Strang

(total of 10 marks)

- Perform the following multiplications (2 marks)
 - $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$
 - $(1 \ 2 \ 3) \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 1 & 4 & 1 \end{pmatrix}$
 - $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & -1 & 1 \end{pmatrix}$

For (c) use the Ways 1 and 4 explained in class.

- Find all matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that (2 marks)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} A = A \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

3. For which three numbers a will elimination fail to give three pivots? Motivate the answer. (2 marks)

$$A = \begin{pmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{pmatrix}.$$

What can you say about matrix A for all possible a 's?

4. Consider the following system of linear equations. (4 marks)

$$\begin{pmatrix} 1 & 3 & 5 \\ -1 & -3 & -1 \\ 3 & 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -5 \\ 6 \end{pmatrix}.$$

- (a) Using elimination and back substitution, find the solution of the system.
(b) Write the elementary matrices E_{21} , E_{31} and E_{32} of the elimination.