

**MATH 3110 - Fall 2015**

**Homework 10**

Due: Thursday November 12

QUESTION 1. *Chapter 6.1 and 6.2 of Strang*

(total of 20 marks)

1. Compute the eigenvalues and the eigenvectors of the following matrices if possible.

(10 marks)

(a)  $A = \begin{pmatrix} 5 & -2 \\ 4 & -1 \end{pmatrix}$

(b)  $B = \begin{pmatrix} -3 & -3 & 6 \\ 6 & 6 & -6 \\ 0 & 0 & 3 \end{pmatrix}$

(c)  $C = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$

(d)  $D = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. Diagonalize matrix  $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$  by finding the matrices  $S$  and  $\Lambda$ .

(5 marks)

3. Diagonalize  $A$  and compute  $S\Lambda^k S^{-1}$  to prove this formula for  $A^k$

(5 marks)

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad \text{and} \quad A^k = \frac{1}{2} \begin{pmatrix} 1 + 3^k & 1 - 3^k \\ 1 - 3^k & 1 + 3^k \end{pmatrix}$$