

# MATH 3110 - Fall 2016

## Homework 1

Due: Thursday, September 1

### Question 1. Chapter 1 of Strang

(total of 10 marks)

1. Four corners of a box are  $(0, 0, 0)$ ,  $(4, 0, 0)$ ,  $(0, 8, 0)$  and  $(0, 0, 6)$ . (3 marks)

- (a) Find the remaining 4 corners.
- (b) Find the coordinates of the center point of the box.
- (c) Find the center points of the six faces.

2. Find two different linear combinations of the vectors  $v_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ ,  $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$  and  $v_3 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$  that produce  $w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . (2 marks)

3. Consider the following three vectors: (3 marks)

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} \text{ and } v_3 = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Find a linear combination of the vectors that give the zero vector.
- (b) Let  $V$  be the  $3 \times 3$  matrix with vectors  $v_1$ ,  $v_2$  and  $v_3$  as columns. What can you say about  $V$ ?
- (c) Where do the three vectors lie in? (line, plane or 3d space) Motivate the answer.
- (d) Following Section 1.2 of Strang, compute the length of the three vectors.

4. Without using elimination, find the solution of the following system of linear equations. (2 marks)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

### Question 2. Chapter 2 of Strang

(total of 10 marks)

1. Perform the following multiplications (3 marks)

(a)  $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

(b)  $(1 \ 2 \ 3) \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 1 & 4 & 1 \end{pmatrix}$

(c)  $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & -1 & 1 \end{pmatrix}$

For (c) use the Ways 1 and 4 explained in class.

2. For which three numbers  $a$  will elimination fail to give three pivots? Motivate the answer. (3 marks)

$$A = \begin{pmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{pmatrix}.$$

What can you say about matrix  $A$  for all possible  $a$ 's?

3. Consider the following system of linear equations. (4 marks)

$$\begin{pmatrix} 1 & 3 & 5 \\ 3 & 3 & 0 \\ -1 & -3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ -5 \end{pmatrix}.$$

- (a) Using elimination and back substitution, find the solution of the system.  
(b) Write the elementary matrices  $E_{21}$ ,  $E_{31}$  and  $E_{32}$  of the elimination.