MATH 3110 - Fall 2016
Homework 7

Due: October 13, 2016

QUESTION 1. *Chapter 4.1 of Strang* *(total of 12 marks)*

1. Find dimension and basis of the orthogonal complement $S^\perp \subset \mathbb{R}^3$ when (4 marks)

   (a) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$ (b) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix} \rangle$

2. Let $P \subseteq \mathbb{R}^4$ be the plane defined the linear equation $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$. (2 marks)
   Write a basis for $P^\perp$ and construct a matrix that has P as nullspace.
   (HINT: write this equation in the form $Ax = 0$.)

3. For each of the following sentences, solve it or motivate if unsolvable. (6 marks)

   (a) Find a matrix with $(1, 4, 2)$ in both its row space and column space.
   (b) Find a matrix with $(1, 4, 2)$ in both its row space and nullspace.
   (c) Find a matrix with $(1, 4, 2)$ in both its column space and nullspace.

QUESTION 2. *Chapter 4.2 of Strang* *(total of 8 marks)*

1. Let $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$ be a line of $\mathbb{R}^3$. Project the vectors $b_1 = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ and $b_2 = \begin{pmatrix} -5 \\ -7 \\ -3 \end{pmatrix}$ onto S. (3 marks)

2. Let $S = \langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rangle$ be a plane in $\mathbb{R}^3$. (5 marks)

   (a) Compute the projection matrix of S.
   (b) Project the following points onto S:

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \; b_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \text{ and } b_3 = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}.$$