

MATH 3110 - Fall 2016

Homework 8

Due: Thursday October 20

QUESTION 1. Chapter 4.2 of Strang

(total of 8 marks)

1. Determine which of the following matrices is a projection matrix (motivate your answer).

For the projection matrices, find the subspace they project onto and its orthogonal complement (give a basis for each of them).

$$(a) A_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$(b) A_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$(c) A_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

QUESTION 2. Chapter 4.3 of Strang

(total of 6 marks)

1. Consider the four data points $(x_i, y_i) = (0, 0), (1, 8), (3, 8)$ and $(4, 20)$.

(a) Find the best fitting line $y = A + Bx$ between the points.

(b) Find the best fitting parabola $y = Cx^2 + Dx + E$ between the points

QUESTION 3. Chapter 4.4 of Strang

(total of 6 marks)

1. Compute using Gram-Schmidt the orthonormal basis of \mathbb{R}^4 related to the following basis vectors

$$v_1 := \begin{pmatrix} 2 \\ 0 \\ 0 \\ -2 \end{pmatrix}, v_2 := \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_3 := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } v_4 := \begin{pmatrix} 0 \\ 2 \\ 0 \\ -2 \end{pmatrix}.$$