MATH 3110 - Fall 2017

Homework 6

Due: October 12, 2017

QUESTION 1. *Chapter 3 of Strang* *(total of 30 marks)*

1. (a) Give a basis for each of the four fundamental subspaces of the matrix (12 marks)

$$A = \begin{pmatrix} 3 & 6 & 2 & -1 & -1 \\ 1 & 2 & 2 & 2 & 11 \\ 0 & 0 & 2 & -3 & 4 \\ 1 & 2 & 0 & 5 & 7 \end{pmatrix}.$$

(b) Determine which of the above subspaces the following vectors belong to: (8 marks)

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 3 \\ -3 \\ 6 \end{pmatrix}, v_4 = \begin{pmatrix} -1 \\ 2 \\ -5 \\ -2 \\ 1 \end{pmatrix}, v_5 = \begin{pmatrix} 0 \\ 4 \\ -4 \\ -4 \end{pmatrix} \text{ and } v_6 = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 2 \\ 14 \end{pmatrix}.$$

2. Determine if the following subsets are subspaces and if they are find their basis. (10 marks)

(a) $V = \langle \begin{pmatrix} 1 \\ 2 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ -1 \end{pmatrix} \rangle$

(b) $V = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mid x_2 = x_1 - x_3 + 1 \right\}$

(c) $V = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \mid x_1 = 0,\ x_2 = x_3 - x_4 \right\}$