**MATH 3110 - Fall 2017**  
**Homework 7**  
**Due:** October 19, 2017

**QUESTION 1. Chapter 4.1 of Strang (total of 18 marks)**

1. Find dimension and basis of the orthogonal complement $S^\perp \subset \mathbb{R}^3$ when  
   (6 marks)
   
   (a) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$
   (b) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \\ -3 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 3 \end{pmatrix} \rangle$

2. Let $P \subseteq \mathbb{R}^4$ be the plane defined the linear equation $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$.  
   Write a basis for $P^\perp$ and construct a matrix that has $P$ as nullspace.  
   (HINT: write this equation in the form $Ax = 0$.)
   (3 marks)

3. For each of the following sentences, solve it or motivate if unsolvable.  
   (9 marks)
   
   (a) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its row space and column space.
   (b) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its row space and nullspace.
   (c) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its column space and nullspace.

**QUESTION 2. Chapter 4.2 of Strang (total of 12 marks)**

1. Let $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$ be a line of $\mathbb{R}^3$. Project the vectors $b_1 = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ and $b_2 = \begin{pmatrix} -5 \\ -7 \\ -3 \end{pmatrix}$ onto $S$.  
   (4 marks)

2. Let $S = \langle \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \rangle$ be a plane in $\mathbb{R}^5$.  
   (8 marks)
   
   (a) Compute the projection matrix of $S$.
   (b) Project the following points onto $S$:  
   $b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $b_3 = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$. 
