MATH 3110 - Fall 2018

Homework 1

Due: Thursday, September 6

QUESTION 1. *Chapter 1 of Strang* *(total of 15 marks)*

test

1. Four corners of a parallelepiped are $\begin{pmatrix}0\\0\\0\end{pmatrix}$, $\begin{pmatrix}10\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\2\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\6\end{pmatrix}$. (3 marks)

   (a) Find the coordinates of the remaining 4 corners.

   (b) Find the coordinates of the center point of the box.

   (c) Find the coordinates of the center points of the six faces.

2. Find two different linear combinations of the vectors $v_1 = \begin{pmatrix}3\\4\end{pmatrix}$, $v_2 = \begin{pmatrix}3\\2\end{pmatrix}$ and $v_3 = \begin{pmatrix}2\\2\end{pmatrix}$ (4 marks)

   that produce $w = \begin{pmatrix}1\\0\end{pmatrix}$.

3. Consider the following three vectors: (6 marks)

$$v_1 = \begin{pmatrix}-1\\2\\-3\end{pmatrix}, v_2 = \begin{pmatrix}2\\-4\\5\end{pmatrix} \text{ and } v_3 = \begin{pmatrix}-1\\2\\-4\end{pmatrix}.$$

   (a) Find a nontrivial linear combination of the vectors that give the zero vector.

   (b) Let V be the 3×3 matrix with vectors v_1, v_2 and v_3 as columns. Find the pivots of V.

   (c) Following Section 1.2 of Strang, compute the length of the three vectors.

4. Without using elimination, find the solution of the following system of linear equations. (2 marks)

$$\begin{pmatrix}1 & 0 & 0 & 0\\2 & -4 & 0 & 0\\1 & -1 & 1 & 0\\1 & -2 & -3 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}2\\0\\1\\1\end{pmatrix}.$$

QUESTION 2. *Chapter 2 of Strang* *(total of 15 marks)*

1. Perform the following multiplications (6 marks)

   (a) $\begin{pmatrix}1 & 3 & 2\\-4 & -2 & 4\\-3 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}$ (b) $\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1 & 3 & 2\\-4 & -2 & 4\\-3 & 1 & 1\end{pmatrix}$ (c) $\begin{pmatrix}1 & 3 & 2\\-4 & -2 & 4\\-3 & 1 & 1\end{pmatrix}\begin{pmatrix}1 & -2 & 3\\4 & -5 & 6\\1 & -1 & 1\end{pmatrix}$

   For (c) use the Ways 1 and 4 explained in class.

2. For which three numbers a will elimination fail to give three pivots? Motivate the answer. (3 marks)

$$A = \begin{pmatrix} a & 3 & -1 \\ a & a & 4 \\ a & a & a \end{pmatrix}.$$

3. Consider the following system of linear equations. (6 marks)

$$\begin{pmatrix} 2 & -1 & 4 \\ -8 & 5 & -13 \\ -4 & 1 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -18 \\ -9 \end{pmatrix}.$$

   (a) Using elimination and back substitution, find the solution of the system.

   (b) Write the elementary matrices E_{21}, E_{31} and E_{32} of the elimination.

PRACTICE QUESTION 1. *Gaussian Elimination*

Perform the Gaussian elimination on the following matrices

1. $\begin{pmatrix} 0 & 0 & 3 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & -12 \end{pmatrix}$ Solution: $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

2. $\begin{pmatrix} 1 & 3 & 4 \\ -1 & -3 & -2 \\ -2 & -6 & -2 \end{pmatrix}$ Solution: $\begin{pmatrix} 1 & 3 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

3. $\begin{pmatrix} 1 & 3 & 2 & -1 & -1 \\ -2 & -6 & -1 & 4 & 3 \\ 2 & 6 & 13 & 6 & 3 \end{pmatrix}$ Solution: $\begin{pmatrix} 1 & 3 & 2 & -1 & -1 \\ 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix}$