QUESTION 1. Chapter 2 of Strang (total of 30 marks)

1. Determine which of the following matrices is invertible and, if invertible, compute the inverse. (8 marks)

\[ A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & -1 \\ 1 & 2 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -3 & -5 \\ 1 & 3 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -6 & -10 \\ 1 & 0 & -1 \end{pmatrix} \]

2. If \( P_1 \) and \( P_2 \) are permutation matrices, so is \( P_1 P_2 \). Give examples of: (6 marks)
   - matrices \( P_1, P_2 \) of size \( 3 \times 3 \) such that \( P_1 P_2 \neq P_2 P_1 \), and
   - matrices \( P_3 \neq P_4 \) of size \( 3 \times 3 \) such the \( P_3 P_4 = P_4 P_3 \) when neither of the matrices is the either identity or the zero matrix.

3. Find the \( A = LU \) factorizations of the following matrix: (6 marks)

\[ A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ -1 & 0 & 3 & 0 \\ 1 & 6 & 0 & 4 \\ 0 & 2 & 1 & 1 \end{pmatrix} \]

4. If \( A \) is a symmetric matrix and \( B \) is an antisymmetric matrix, which of the following matrices is symmetric? (Motivate the answer) (6 marks)
   - (a) \( A^3 - B^3 \)
   - (b) \( ABAB \)
   - (c) \( (A + B)(A - B) \)

5. (a) Let \( A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \). Find matrices \( B, C \) such that \( A = B + C \) with \( B = B^T \) (symmetric), and \( C = -C^T \) (anti-symmetric). (2 marks)

   \[ B = B^T \] 
   \[ C = -C^T \]

(b) Find formulas for \( B \) and \( C \) involving \( A \) and \( A^T \). We want \( A = B + C \), \( B = B^T \) and \( C = -C^T \). (2 marks)