Homework 2

Due: Thursday September 13

QUESTION 1. *Chapter 2 of Strang* *(total of 30 marks)*

1. Determine which of the following matrices is invertible and, if invertible, compute the inverse. (8 marks)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & -1 \\ 1 & 2 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -3 & -5 \\ 1 & 3 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -6 & -10 \\ 1 & 0 & -1 \end{pmatrix}$$

2. If P_1 and P_2 are permutation matrices, so is $P_1 P_2$. Give examples of: (6 marks)

   - matrices P_1, P_2 of size 3×3 such that $P_1 P_2 \neq P_2 P_1$, and
   - matrices $P_3 \neq P_4$ of size 3×3 such the $P_3 P_4 = P_4 P_3$ when neither of the matrices is the either identity or the zero matrix.

3. Find the $A = LU$ factorizations of the following matrix: (6 marks)

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ -1 & 0 & 3 & 0 \\ 1 & 6 & 0 & 4 \\ 0 & 2 & 1 & 1 \end{pmatrix}$$

4. If A is a symmetric matrix and B is an antisymmetric matrix, which of the following matrices is symmetric? (Motivate the answer) (6 marks)

   (a) $A^3 - B^3$ (b) $ABAB$ (c) $(A + B)(A - B)$

5. (a) Let $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$. Find matrices B, C such that $A = B + C$ with (2 marks)

$$B = B^T \text{ (symmetric), and } C = -C^T \text{ (anti-symmetric)}.$$

   (b) Find formulas for B and C involving A and A^T. We want $A = B + C$, $B = B^T$ and $C = -C^T$. (2 marks)