MATH 3110 - Fall 2018
Homework 3

Due: Thursday September 20

ALWAYS MOTIVATE THE ANSWERS!

QUESTION 1. *Chapter 3.1 of Strang* *(total of 30 marks)*

1. Which of the following subsets of $\mathbb{R}^3$ are actually subspaces? (Motivate the answers) (8 marks)

   (a) $S_1 = \left\{ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \middle| b_2 = b_1 + b_3 \right\}$.

   (c) $S_3 = \left\{ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \middle| b_1 b_2 b_3 = 0 \right\}$.

   (b) $S_2 = \left\{ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \middle| b_3 = b_1 = 2 \right\}$.

   (d) $S_4 =$ all linear combinations of $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$.

2. Show that the following subsets V of $\mathbb{R}^3$ are not subspaces, meaning find a counterexample to one (6 marks)
   of the main properties.

   (a) $V = P \setminus L = \{x \in \mathbb{R}^3 \mid x \in P \text{ and } x \notin L\}$, meaning all the elements of P that are
   not elements of L, where P is a plane passing through 0 and L is a line passing through 0 in $\mathbb{R}^3$.

   (b) $V = \{x \in \mathbb{R}^3 \mid \|x\| = 1\}$ where $\|x\|$ represents the length of a vector $x \in \mathbb{R}^3$.
   (Hint: V is a sphere of radius 1.)

   (c) $V = \{x \in \mathbb{R}^3 \mid \|x\| \leq 1\}$.
   (Hint: V is a ball of radius 1.)

3. Consider the following matrices (7 marks)

   $$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}.$$

   (a) Show that both matrices are singular?

   (b) Explicitly write using mathematical notation their column spaces, $C(A)$ and $C(B)$?

   (c) Show that the columns of B are elements of $C(A)$.

   (d) Show that the columns of A are elements of $C(B)$.

4. Construct a 3×3 matrix whose column space contains only the vector $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. (3 marks)

5. Construct a 3×3 matrix whose column space contains vectors $\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and not $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (3 marks)

6. Construct a 3×3 matrix whose column space is a line. (3 marks)