MATH 3110 - Fall 2018

Homework 4

Due: Thursday September 27

QUESTION 1. *Chapter 3 of Strang* *(total of 30 marks)*

1. Compute the row reduced echelon form of the following matrices (6 marks)

$$A = \begin{pmatrix} 1 & 2 & 2 & 3 & 9 \\ 3 & 6 & 1 & 4 & 7 \\ 0 & 0 & 1 & 1 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 3 \\ 4 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

2. Construct a matrix A such that $N(A)$ contains all multiples of $\begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$. (2 marks)

3. (a) Write the 3×7 matrix in rref with the largest amount of entries equal 1. (2 marks)
   (b) Write the 3×7 matrix in rref with the largest amount of entries equal 1 and pivot columns 2 and 4. (2 marks)

4. Answer the following questions. (6 marks)
   (a) Find a matrix A such that the only solution of $Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
   (b) Show that it is not possible to find a matrix B such that the *only* solution of $Bx = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

5. Compute rank and set of solutions (by finding a particular solution and the nullspace) of the systems: (12 marks)

1. $$\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ 5 \end{pmatrix}$$

2. $$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

3. $$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 10 \end{pmatrix}$$

4. $$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$$