MATH 3110 - Fall 2018
Homework 7

Due: October 18, 2018

QUESTION 1. *Chapter 4.1 of Strang* *(total of 20 marks)*

1. Find dimension and basis of the orthogonal complement $S^{\perp} \subset \mathbb{R}^3$ of the following spaces (8 marks)

   (a) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$

   (b) $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix} \rangle$

2. Consider the subspace (3 marks)

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \middle| \, x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \right\}.$$

   Construct a matrix A such that $P = N(A)$ and write a basis for the orthogonal complement $S^{\perp}$.

3. For each of the following sentences, solve it or motivate if unsolvable. (9 marks)

   (a) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its row space and column space.

   (b) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its row space and nullspace.

   (c) Find a matrix with $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ in both its column space and nullspace.

QUESTION 2. *Chapter 4.2 of Strang* *(total of 10 marks)*

1. Let $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$ be a line of $\mathbb{R}^3$. (10 marks)

   (a) Projects vectors $b_1 = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$ and $b_2 = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$ onto S.

   (b) Compute the projection matrix P of S and project the vectors $b_3 = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$, $b_4 = \begin{pmatrix} -5 \\ -7 \\ -3 \end{pmatrix}$ onto S with it.