MATH 3110 - Fall 2018

Homework 8

Due: Thursday October 25

QUESTION 1. *Chapter 4.2 of Strang* *(total of 16 marks)*

1. Let $S = \langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rangle$ be a plane in $\mathbb{R}^3$. (6 marks)

   (a) Compute the projection matrix of S.

   (b) Project the following points onto S:

$$b_1 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}, \; b_2 = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} \text{ and } b_3 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$

2. Determine if the following matrices iare projection matrices (motivate your answer). (10 marks)
   For the projection matrices, find the subspace they project onto and its orthogonal complement (give a basis for each of them).

   (a) $A_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ (b) $A_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ (c) $A_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$

QUESTION 2. *Chapter 4.3 of Strang* *(total of 8 marks)*

1. Consider the four data points $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 20 \end{pmatrix}$.

   (a) Find the best fitting line $y = \alpha + \beta x$ between the points.

   (b) Find the best fitting parabola $y = \gamma x^2 + \delta x + \epsilon$ between the points

QUESTION 3. *Chapter 4.4 of Strang* *(total of 6 marks)*

1. Compute using Gram-Schmidt the orthonormal basis of $\mathbb{R}^4$ related to the following basis vectors

$$v_1 := \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \; v_2 := \begin{pmatrix} 3 \\ 0 \\ 0 \\ -3 \end{pmatrix}, \; v_3 := \begin{pmatrix} -2 \\ -2 \\ -2 \\ -2 \end{pmatrix} \text{ and } v_4 := \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}.$$