MATH 3110 - Spring 2014

Homework 1

Due: Jan. 28th (Tuesday)

Question 1. Chapter 1 of Strang **(total of 10 marks)**

1. Four corners of a rectangle are $(0, 0, 0)$, $(2, 0, 0)$, $(0, 4, 0)$ and $(0, 0, 10)$. (3 marks)
   (a) Find the remaining 4 corners.
   (b) Find the coordinates of the center point of the rectangle.
   (c) Find the center points of the six faces.

2. Find two different linear combinations of the vectors $v_1 = (4, 3)$, $v_2 = (1, 1)$ and $v_3 = (5, 4)$ that produce $w = (1, 0)$. (2 marks)

3. Consider the following three vectors: (3 marks)

$$v_1 = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix} \text{ and } v_3 = \begin{pmatrix} 7 \\ -3 \\ 10 \end{pmatrix}.$$

   (a) Find a linear combination of the vectors that give the zero vector.
   (b) Let V be the 3×3 with vectors v_1, v_2 and v_3 as columns. What can you say about V?
   (c) Where do the three vectors lie in? (line, plane or 3d space) Motivate the answer.
   (d) Following Section 1.2 of Strang, compute the length of the three vectors.

4. Without using elimination, find the solution of the following system of linear equations. (2 marks)

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \\ 2 \end{pmatrix}.$$

Question 2. Chapter 2 of Strang **(total of 10 marks)**

1. Perform the following multiplications (2 marks)

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 10 & 6 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 10 & 6 \\ 1 & 4 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

   For (c) use the Ways 1 and 4 explained in class.

2. Find all matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that (2 marks)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} A = A \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

3. For which three numbers a will elimination fail to give three pivots? Motivate the answer. (2 marks)

$$A = \begin{pmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{pmatrix}.$$

   What can you say about matrix A for all possible a's?

4. Consider the following system of linear equations. (4 marks)

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

(a) Using elimination and back substitution, find the solution of the system.

(b) Write the elementary matrices E_{21}, E_{31} and E_{32} of the elimination.

(c) If A is the matrix related to the system, compute A^{-1} using Gauss-Jordan, and show that the solution is $A^{-1}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(d) Find the matrix L of the LU-decomposition of A.