MATH 3110 - Spring 2014

Homework 5

Due: Feb. 27th (Thursday)

Questions. Chapter 3 of Strang **(total of 20 marks)**

1. Find the dimension and a basis of the following subspaces of the space of $n \times n$ matrices. (6 marks)
   (a) Lower triangular matrices.
   (b) All symmetric matrices.
   (c) All anti-symmetric matrices.

2. Set of real polynomials. (6 marks)
   (a) Prove that the set
   $$\mathbb{R}[x] = \{p(x) = p_0 + p_1 x + \cdots + p_d x^d \mid p_0, \ldots, p_d \in \mathbb{R},\ d \in \mathbb{N}\},$$
   i.e., the set of the polynomials with real coefficients is a vector space.
   (b) Prove that the set of polynomials with degree less than or equal to 3 is a subspace of $\mathbb{R}[x]$. Find the dimension and a basis of it.
   (c) Same for the set $\{p(x) \in \mathbb{R}[x] \mid p(1) = 0\}$.

3. Let $V, W \in \mathbb{R}^n$. Prove that if $\dim(V) + \dim(W) > n$, than there exists a nonzero $v \in \mathbb{R}^n$ such that (2 marks)
   $$v \in V \cap W.$$

4. Without computing A, find bases for its four fundamental subspaces. (4 marks)
   $$A = \begin{pmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 9 & 8 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix}.$$

5. Without computing A, find bases for the row and column space. (2 marks)
   $$A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{pmatrix} \begin{pmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}.$$