

# MATH 3110 - Spring 2014

## Homework 6

Due: Mar. 6th (Thursday)

### Questions. Chapter 3.1 of Strang

(total of 20 marks)

1. Find dimension and basis of the space  $S^\perp \subset \mathbb{R}^3$  when (6 marks)

(a)  $S = \{0\}$

(b)  $S = \left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\rangle$

(c)  $S = \left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix} \right\rangle$

2. Let  $P \subseteq \mathbb{R}^4$  be the plane defined the linear equation  $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$ . (2 marks)  
Write a basis for  $P^\perp$  and construct a matrix that has  $P$  as nullspace.

3. Find  $A^T A$  if the columns of  $A$  are unit vectors and all mutually perpendicular. (4 marks)

4. For each of the following sentences, solve it or motivate if unsolvable. (6 marks)

(a) Find a matrix with  $(1, 4, 2)$  in both its row space and column space.

(b) Find a matrix with  $(1, 4, 2)$  in both its row space and nullspace.

(c) Find a matrix with  $(1, 4, 2)$  in both its column space and nullspace.

5. Let  $A$  be an  $n \times m$  matrix. Prove that  $((C(A^T))^\perp)^\perp = R(A)$ . (2 marks)