Question 1. Chapter 4.2 of Strang

1. Let $S = \langle (1 \ 2 \ 3)^T \rangle$ be a line of $\mathbb{R}^3$. Project the vectors $(5 \ 7 \ 3)^T$ and $(-5 \ -7 \ -3)^T$ onto $S$. (3 marks)

2. Consider the subset $S \subseteq \mathbb{R}^4$ defined by the equation $x - y - 2z = 0$. (6 marks)
   
   (a) Find the dimension of $S$ and give a basis of it.
   
   (b) Consider the basis to be the columns of a matrix $A_1$ such that $S = C(A_1)$. Compute the projection matrix $P_1$ for $S$.
   
   (c) Find another basis for $S$ and compute the projection matrix $P_2$. Notice that $P_1 = P_2$, meaning that the projection matrix does not depend on the choice of the basis.

3. Show that if $P$ is a projection matrix, then $I - P$ is a projection matrix. (2 marks)

4. Let $S \subseteq \mathbb{R}^n$. Show that for every vector $v \in \mathbb{R}^n$ there exist two vectors $v_S \in S$ and $v_{S^\perp} \in S^\perp$ such that $v = v_S + v_{S^\perp}$. (3 marks)

Question 2. Chapter 4.3 of Strang

1. Consider the four data points $(t_i, b_i) = (0, 0), (1, 8), (3, 8)$ and $(4, 20)$. Find the best fitting line $b = C + Dt$ between the points. (6 marks)