

# MATH 3110 - Spring 2014

## Homework 8

Due: Mar. 27th (Thursday)

### Question 1. Chapter 4.4 of Strang

(total of 10 marks)

1. Find a basis of the plane  $2x + y - 2z = 0$  and make it orthonormal. (3 marks)

2. Using Gram-Schmidt, make the vectors  $a_1 = \begin{pmatrix} 2 \\ 3 \\ 6 \\ 0 \end{pmatrix}$ ,  $a_2 = \begin{pmatrix} 2 \\ 1 \\ 7 \\ 2 \end{pmatrix}$  and  $a_3 = \begin{pmatrix} -1 \\ 3 \\ 7 \\ 4 \end{pmatrix}$  orthonormal. (3 marks)

3. Let us prove that orthonormal vectors  $q_1, q_2, q_3$  are linearly independent. (4 marks)

(a) Vector version. Using vector product, prove that if  $c_1q_1 + c_2q_2 + c_3q_3 = 0$  then  $c_1 = c_2 = c_3 = 0$ .

(Hint: Which vector product leads to  $c_i = 0$  for  $i = 1, 2, 3$ ?)

(b) Matrix version: Show that  $Qx = 0$  leads to  $x = 0$ .

### Question 2. Chapter 5.1 of Strang

(total of 10 marks)

1. Let  $A$  be a  $4 \times 4$  matrix with determinant 3. Compute the determinant of the following matrices. (3 marks)

(a)  $(3A)^{-1}$

(b)  $((A + A)^T - A^T) * A$

2. Using only properties from Section 5.1, compute the determinant of the following matrices with respect to the parameter  $\lambda$ . For which values of  $\lambda$  are the matrices singular? (4 marks)

(a)  $\begin{pmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & 3 \\ \lambda & \lambda & \lambda \end{pmatrix}$ .

(b)  $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} - \lambda I$  where  $I$  is the  $2 \times 2$  identity matrix.

3. If you know that  $\det(A) = \begin{vmatrix} \text{row1} \\ \text{row2} \\ \text{row3} \end{vmatrix} = 6$ , what is the determinant of  $B = \begin{pmatrix} \text{row3} + \text{row2} + \text{row1} \\ \text{row2} + \text{row1} \\ \text{row1} \end{pmatrix}$ ? (3 marks)