MATH 3110 - Spring 2014

Homework 11

Due: Apr. 17th (Thursday)

Question. Chapter 6.3 of Strang **(total of 10 marks)**

1. (a) Prove that if every column of A adds to zero, then $\lambda = 0$ is an eigenvalue of A. (4 marks)

   (b) Find the general solutions of the system

$$\frac{du}{dt} = \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix} u \quad \text{with} \quad u(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

2. Let $A = \begin{pmatrix} a & 1 \\ 1 & a \end{pmatrix}$ and $B = \begin{pmatrix} b & -1 \\ 1 & b \end{pmatrix}$. Find the conditions on a and b such that all solutions of (3 marks)

$$\frac{du}{dt} = Au \quad \text{and} \quad \frac{dv}{dt} = Bv$$

   approach to zero as $t \to \infty$.

3. Compute e^A for $A = \begin{pmatrix} -3 & -2 & -1 \\ 6 & 4 & 2 \\ 4 & 2 & 2 \end{pmatrix}$. (3 marks)