Question. Chapter 6.3 of Strang (total of 10 marks)

1. (a) Prove that if every column of $A$ adds to zero, then $\lambda = 0$ is an eigenvalue of $A$. (4 marks)
   
   (b) Find the general solutions of the system
   
   $$\frac{du}{dt} = \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix} u \quad \text{with} \quad u(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

2. Let $A = \begin{pmatrix} a & 1 \\ 1 & a \end{pmatrix}$ and $B = \begin{pmatrix} b & -1 \\ 1 & b \end{pmatrix}$. Find the conditions on $a$ and $b$ such that all solutions of $\frac{du}{dt} = Au$ and $\frac{dv}{dt} = Bv$ approach to zero as $t \to \infty$. (3 marks)

3. Compute $e^A$ for $A = \begin{pmatrix} -3 & -2 & -1 \\ 6 & 4 & 2 \\ 4 & 2 & 2 \end{pmatrix}$. (3 marks)