

MATH 3110 - Spring 2014

Homework 12

Due: Apr. 24th (Thursday)

Question. Chapter 6.4-5-6-7 of Strang

(total of 20 marks)

1. (a) Find an orthogonal matrix Q that diagonalizes $A = \begin{pmatrix} -2 & 6 \\ 6 & 7 \end{pmatrix}$. What is Λ ? (4 marks)

(b) Find all orthogonal matrices that diagonalize $A = \begin{pmatrix} 9 & 12 \\ 12 & 16 \end{pmatrix}$.

2. Show that A and B are similar finding M such that $B = M^{-1}AM$. (6 marks)

• $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

• $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

• $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

3. The following Jordan matrices have eigenvalues $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ (4 marks)

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad K = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Show that these matrices are not similar, i.e., that if there exists a matrix M such that $JM = MK$, then M is not invertible.

4. Compute the SVD of the matrix

(6 marks)

$$A = \begin{pmatrix} \frac{\sqrt{2}}{2} & -1 & 1 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -1 & 1 \end{pmatrix}$$