MATH 8530 - Fall 2015

Homework 4

Due: Oct. 27 (Tuesday)

QUESTION 1.

Find the eigenvalues and corresponding eigenvectors for the following matrices over $\mathbb{C}$.

$$(a) \quad \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix} \qquad (b) \quad \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix} \qquad (c) \quad \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}.$$

QUESTION 2.

1. Show that if A and B are similar, then A and B have the same eigenvalues.
2. Is the converse of Part (a) true? Prove or disprove.

QUESTION 3.

Let A_θ be a 3×3 matrix representing a rotation of $\mathbb{R}^3$ through an angle θ about the y-axis.

1. Find the eigenvalues for A_θ over $\mathbb{C}$.
2. Determine necessary and sufficient conditions on θ in order for A_θ to contain three linearly independent eigenvectors in $\mathbb{R}^3$. Justify your claim and interpret it geometrically.

QUESTION 4.

Let A be a 2×2 matrix over $\mathbb{R}$ satisfying $A^T = A$. Prove that A has 2 linearly independent eigenvectors in $\mathbb{R}^2$.

QUESTION 5.

Let A be an invertible $n \times n$ matrix. Prove that A^{-1} can be written as a polynomial in degree at most $n - 1$. That is, prove that there are scalars c_i such that

$$A^{-1} = c_{n-1}A^{n-1} + c_{n-2}A^{n-2} + \cdots c_1 A + c_0 I .$$