MATH 8560 - Spring 2014

Homework 4

<u>Due</u>: Mar. 15 (Tuesday)

Question 1. **(2 marks)**

(Perfect codes)

Let $\mathcal{C}$ be a perfect $(n, M, d = 2t+1)$ code over $\mathbb{F}_q$ and suppose that $0 \in \mathcal{C}$. Show that the cardinality of

$$|\{c \in \mathcal{C} \mid \mathrm{wt}(c) = 2t+1\}| = \frac{\binom{n}{t+1}(q-1)^{t+1}}{\binom{2t+1}{t}}.$$

Question 2. **(4 marks)**

(Constant weight codes)

Let $\mathcal{C}$ be a $(n, M, d; w)$ constant-weight code, i.e., a code having only codewords of weight w.

- Prove or disprove: there exists a linear constant-weight code.
- If $\mathcal{C}$ has parameters $(n, M, 2t+1; 2t+1)$, prove that

$$M \leq \frac{\binom{n}{t+1}(q-1)^{t+1}}{\binom{2t+1}{t}}.$$

- When is the bound attained?

Question 3. **(2 marks)**

(Doubly-extended GRS codes)

A $[n, n-r, d]$ code $\mathcal{C}$ over $\mathbb{F}$ is a doubly-extended GRS code if it is defined by a parity check matrix

$$H = \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_{n-1}^2 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ \alpha_1^{r-2} & \alpha_2^{r-2} & \cdots & \alpha_{n-1}^{r-2} & 0 \\ \alpha_1^{r-1} & \alpha_2^{r-1} & \cdots & \alpha_{n-1}^{r-1} & 1 \end{pmatrix} \begin{pmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & v_n \end{pmatrix}$$

where $\alpha_1, \ldots, \alpha_{n-1}$ are distinct elements and $v_1, \ldots, v_n$ are nonzero elements of $\mathbb{F}_q$.

- Show that $\mathcal{C}$ is MDS.
- Show that $\mathcal{C}^\perp$ is also a doubly-extended GRS code.

Question 4. **(7 marks)**

(Decoding errors and erasures for GRS codes)

Solve Problem 6.11, page 207 of Roth. (Yes, I know!)

Enjoy the Spring break!