22.1 (a) \( g \circ f \) is defined and is equal to \( \{(1, 1), (2, 1), (3, 1)\} \). \( f \circ g \) is defined and is equal to \( \{(2, 2), (3, 2), (4, 2)\} \). \( f \circ g \neq g \circ f \).
(b) \( g \circ f \) is defined and is equal to \( \{(1, 1), (2, 2), (3, 3)\} \). \( f \circ g \) is defined and is equal to \( \{(2, 2), (3, 3), (4, 4)\} \). \( f \circ g \neq g \circ f \).
(c) \( g \circ f \) is defined and is equal to \( \{(1, 0), (2, 5), (3, 3)\} \). \( f \circ g \) is defined and is equal to \( \{(2, 2), (3, 3), (4, 4)\} \). \( f \circ g \neq g \circ f \).
(f) \( g \circ f \) is defined and is equal to \( (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 2 \). \( f \circ g \) is also defined. It is equal to \( (x^2 + 1)^2 - 1 = x^4 + 2x^2 \). So \( f \circ g \neq g \circ f \).
(g) \( g \circ f \) is defined and equals \( (x + 3) - 7 = x - 4 \). \( f \circ g \) is defined and equals \( (x - 7) + 3 = x - 4 \). So \( f \circ g = g \circ f \).

31.1
(a) \( q = 33 \) and \( r = 1 \).
(b) \( q = -34 \) and \( r = 2 \).
(c) \( q = 33 \) and \( r = 0 \).
(d) \( q = -33 \) and \( r = 0 \).
(e) \( q = 0 \) and \( r = 0 \).

31.2
(a) \( 100 \div 3 = 33 \) and \( 100 \mod 3 = 1 \).
(b) \( -100 \div 3 = -34 \) and \( -100 \mod 3 = 2 \).
(c) \( 99 \div 3 = 33 \) and \( 99 \mod 3 = 0 \).
(d) \( -99 \div 3 = -33 \) and \( -99 \mod 3 = 0 \).
(e) \( 0 \div 3 = 0 \) and \( 0 \mod 3 = 0 \).

31.9
(a) Let \( p \) and \( q \) be polynomials. We say that \( p \) divides \( q \) (and we write \( p | q \)) provided there is a polynomial \( r \) so that \( q = pr \). There is a typo in the book for the second part of this question: It should read “Why is \( (x - 3)(x^3 - 3x^2 + 3x - 9) \)?” The answer is: “because \( x^3 - 3x^2 + 3x - 9 = (x - 3)(x^2 + 3) \).
(b) Let \( p = x + 1 \) and let \( q = 2x + 2 \). Then \( p | q \) because \( q = (2)p \) and \( q | p \) because \( p = \frac{1}{2}q \).
(c) \( p | q \) and \( q | p \) iff there is a nonzero rational number \( r \) so that \( p = rq \).