MthSc 119, 22.1abcdg, 31.1abcde, 31.2, 31.9 abc

22.1 (a) $g \circ f$ is defined and is equal to $\{(1,1),(2,1),(3,1)\}$. $f \circ g$ is defined and is equal to $\{(2,2),(3,2),(4,2)\}$. $f \circ g \neq g \circ f$.

(b) $g \circ f$ is defined and is equal to $\{(1,1),(2,2),(3,3)\}$. $f \circ g$ is defined and is equal to $\{(2,2),(3,3),(4,4)\}$. $f \circ g \neq g \circ f$.

(c) $g \circ f$ is defined and is equal to $\{(1,0),(2,5),(3,3)\}$. $f \circ g$ is not defined since $0 \in \operatorname{im} g$ but $0 \notin \operatorname{dom} f$.

(f) $g \circ f$ is defined and is equal to $(x^2-1)^2+1 = x^4-2x^2+2$. $f \circ g$ is also defined. It is equal to $(x^2+1)^2-1 = x^4+2x^2$. So $f \circ g \neq g \circ f$.

(g) $g \circ f$ is defined and equals $(x+3)-7 = x-4$. $f \circ g$ is defined and equals $(x-7)+3 = x-4$. So $f \circ g = g \circ f$.

31.1
(a) $q = 33$ and $r = 1$.
(b) $q = -34$ and $r = 2$.
(c) $q = 33$ and $r = 0$.
(d) $q = -33$ and $r = 0$.
(e) $q = 0$ and $r = 0$.

31.2
(a) $100 \operatorname{div} 3 = 33$ and $100 \bmod 3 = 1$.
(b) $-100 \operatorname{div} 3 = -34$ and $-100 \bmod 3 = 2$.
(c) $99 \operatorname{div} 3 = 33$ and $99 \bmod 3 = 0$.
(d) $-99 \operatorname{div} 3 = -33$ and $-99 \bmod 3 = 0$.
(e) $0 \operatorname{div} 3 = 0$ and $0 \bmod 3 = 0$.

31.9
(a) Let p and q be polynomials. We say that p divides q (and we write $p|q$) provided there is a polynomial r so that $q = pr$. There is a typo in the book for the second part of this question: It should read "Why is $(x-3)|(x^3-3x^2+3x-9)$." The answer is: "because $x^3-3x^2+3x-9 = (x-3)(x^2+3)$.

(b) Let $p = x+1$ and let $q = 2x+2$. Then $p|q$ because $q = (2)p$ and $q|p$ because $p = \frac{1}{2}q$.

(c) $p|q$ and $q|p$ iff there is a nonzero rational number r so that $p = rq$.