MthSc 119, EXTRA PRACTICE

32.1 (a) $\gcd(20,25)=5$.
(b) $\gcd(0,10)=10$.
(c) $\gcd(123,-123)=123$.
(d) Here we use the Euclidean Algorithm, noting first that $\gcd(-89,-98)=\gcd(89,98)$.:

$$\begin{aligned} 98 &= 1\times 89+9 \\ 89 &= 9\times 9+8 \\ 9 &= 1\times 8+1 \\ 8 &= 8\times 1+0 \end{aligned}$$

We conclude that $\gcd(89,98)=1$.
(e) Here we use the Euclidean Algorithm:

$$\begin{aligned} 54321 &= 1086\times 50+21 \\ 50 &= 2\times 21+8 \\ 21 &= 2\times 8+5 \\ 8 &= 1\times 5+3 \\ 5 &= 1\times 3+2 \\ 3 &= 1\times 2+1 \\ 2 &= 2\times 1+0 \end{aligned}$$

We conclude that $\gcd(54321,50)=1$. Unless you're required to use the Euclidean Algorithm here, though, it is not hard to see that the only prime divisors of 50 are 2 and 5, neither of which are divisors of 54321, hence the greatest common divisor of 50 and 54321 must be 1.
(f)

$$\begin{aligned} 29341 &= 16\times 1739+1517 \\ 1739 &= 1\times 1517+222 \\ 1517 &= 6\times 222+185 \\ 222 &= 1\times 185+37 \\ 185 &= 5\times 37+0 \end{aligned}$$

We conclude that $\gcd(29341, 1739) = 37$.

32.2

(d) Using back substitution together with our work above for question 32.1 (d), we get

$$\begin{aligned} 1 &= 9 - 1 \times 8 \\ &= 9 - (89 - 9 \times 9) \\ &= 10 \times 9 - 89 \\ &= 10(98 - 89) - 89 \\ &= 10 \times 98 - 11 \times 89 \end{aligned}$$

So $1 = 10 \times 98 - 11 \times 89$.

(e) Using back substitution together with our work above for question 32.1 (e), we get

$$\begin{aligned} 1 &= 3 - 2 \\ &= 3 - (5 - 3) \\ &= 2 \times 3 - 5 \\ &= 2(8 - 5) - 5 \\ &= 2 \times 8 - 3 \times 5 \\ &= 2 \times 8 - 3(21 - 2 \times 8) \\ &= 8 \times 8 - 3 \times 21 \\ &= 8(50 - 2 \times 21) - 3 \times 21 \\ &= 8 \times 50 - 19 \times 21 \\ &= 8 \times 50 - 19(54321 - 1086 \times 50) \\ &= -19(54321) + 20642(50) \end{aligned}$$

Therefore $1 = -19(54321) + 20642(50)$.

33.1

(a) 6

(b) 2

(c) 0

(d) 7

(e) undefined because $12 \notin \mathbb{Z}_{10}$.

Other practice problems.
1. Find 8^{-1} in $\mathbb{Z}_{25}$. Show your work.

$$\begin{aligned} 25 &= 3 \times 8 + 1 \\ 8 &= 8 \times 1 + 0 \end{aligned}$$

Back-substitution, which in this case is very simple, gives us $1 = 25 - 3 \times 8$. It follows that $8^{-1} = 3$.

2. Which integers in $\mathbb{Z}_{26}$ have inverses?
$1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25$.

4. Alice uses the Caesar Cypher and sends Bob the message below. What is she saying?

$$PHUUBFKULVWPDV$$

MERRY CHRISTMAS