MthSc 119, Section 12

12.1 (a) Yes. (b) No. (c) No. (d) No. (e) Yes. (f) No.

12.2 Let x and y be odd integers. Since x and y are odd there are integers a and b with $x = 2a+1$ and $y = 2b+1$. Observe that $x-y = (2a+1)-(2b+1) = 2(a-b)$. Thus $2|(x-y)$. Therefore $x \equiv y \pmod 2$. QED

Let x and y be even integers. Thus we can find integers a and b with $x = 2a$ and $y = 2b$. Note that $x - y = 2a - 2b = 2(a - b)$. So $2|(x - y)$. Thus $x \equiv y \pmod 2$. QED

12.4 Let x, y and z be integers with $x \equiv y$ and $y \equiv z$. This means that $n|(x-y)$ and $n|(y-z)$. Thus there are integers a and b with $x-y = na$ and $y-z = nb$. Note that $x-z = (x-y)+(y-z) = na+nb = n(a+b)$. Hence $n|(x - z)$, showing that $x \equiv z$. QED

12.5 (a) $[1] = \{1, 2\}$.
(b) $[4] = \{4\}$.
(d) [you] = the set containing you and all your siblings.
(f) $[\{1, 3\}] = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}\}$.

12.7 Let R be an equivalence relation on a set A and let $a, b \in A$.
($\Rightarrow$) Assume that $a \in [b]$. Since $[b] = \{x \in A : xRa\}$, this means that aRb. Since R is an equivalence relation, R is symmetric. Hence, bRa. Therefore $b \in [a]$.

($\Leftarrow$) A similar argument shows that if $b \in [a]$ then $a \in [b]$. QED