MthSc 119, Assignment 3 — Model Solutions

3.2 (2 points) We will show that if x is an even integer and y is an odd integer, then $x+y$ is odd. Let x be an even integer and let y be an odd integer. By definition of even, we know that $2|x$. Hence there is an integer a with $x = 2a$. Since y is odd we know, again by definition, that there is an integer b with $y = 2b + 1$. Now $x + y = 2a + (2b + 1) = (2a + 2b) + 1 = 2(a + b) + 1$. Hence it follows, by the definition of odd, that $x + y$ is odd. QED

3.3 (2 points) We will show that if x and y are even integers then $x \cdot y$ is even. Let x and y be even integers. Therefore there are integers a and b with $x = 2a$ and $y = 2b$. Now $x \cdot y = (2a)(2b) = 2(2ab)$ and so $x \cdot y$ is even. QED

3.6 (2 points) We will show that if a, b and c are integers with $a|b$ and $a|c$, then $a|(b+c)$. Let a, b and c be integers with $a|b$ and $a|c$. By definition there are integers x and y with $b = ax$ and $c = ay$. Therefore $b + c = ax + ay = a(x + y)$. Hence there is an integer z, namely $x + y$, with $b + c = az$ and so $b + c$ is divisible by a, i.e. $a|(b + c)$. QED

3.10 (2 points) We will show that x is an odd integer if and only if $x + 1$ is an even integer.

($\Rightarrow$) Let x be an odd integer. Since x is odd there is an integer a with $x = 2a + 1$. Note that $x + 1 = (2a + 1) + 1 = 2a + 2 = 2(a + 1)$. Therefore $x + 1$ is even.

($\Leftarrow$) Now suppose that $x + 1$ is even. Therefore $2|(x + 1)$ and hence there is an integer b with $x + 1 = 2b$. Subtracting 1 from both sides of this equation, we get $x = 2b - 1 = 2(b - 1) + 1$ and so x is odd. QED

3.11 (2 points) Let x be an integer. We will show that $0|x$ if and only if $x = 0$.

($\Rightarrow$) Suppose $0|x$. Since 0 is a divisor of x there is an integer a with $x = 0 \cdot a$. Therefore $x = 0 \cdot a = 0$.

($\Leftarrow$) Because $0 \cdot 1 = 0$, we know that $0|0$. QED

3.13 (2 points) Let x be an integer. We will show that x is odd if and only if x is the sum of two consecutive integers.

($\Rightarrow$) Suppose x is odd. By definition, there is an integer a with $x = 2a+1 = a+(a+1)$. It follows that x is the sum of two consecutive integers, namely a and $a+1$.

($\Leftarrow$) Now suppose that x is the sum of two consecutive numbers, say b and $b+1$. Then $x = b+(b+1) = 2b+1$, showing that x is odd. QED