3.2 (2 points) We will show that if \( x \) is an even integer and \( y \) is an odd integer, then \( x+y \) is odd. Let \( x \) be an even integer and let \( y \) be an odd integer. By definition of even, we know that \( 2 \mid x \). Hence there is an integer \( a \) with \( x = 2a \). Since \( y \) is odd we know, again by definition, that there is an integer \( b \) with \( y = 2b + 1 \). Now \( x + y = 2a + (2b + 1) = (2a + 2b) + 1 = 2(a + b) + 1 \). Hence it follows, by the definition of odd, that \( x + y \) is odd. QED

3.3 (2 points) We will show that if \( x \) and \( y \) are even integers then \( x \cdot y \) is even. Let \( x \) and \( y \) be even integers. Therefore there are integers \( a \) and \( b \) with \( x = 2a \) and \( y = 2b \). Now \( x \cdot y = (2a)(2b) = 2(2ab) \) and so \( x \cdot y \) is even. QED

3.6 (2 points) We will show that if \( a \), \( b \) and \( c \) are integers with \( a \mid b \) and \( a \mid c \), then \( a \mid (b+c) \). Let \( a \), \( b \) and \( c \) be integers with \( a \mid b \) and \( a \mid c \). By definition there are integers \( x \) and \( y \) with \( b = ax \) and \( c = ay \). Therefore \( b + c = ax + ay = a(x + y) \). Hence there is an integer \( z \), namely \( x + y \), with \( b + c = az \) and so \( b + c \) is divisible by \( a \), i.e. \( a \mid (b + c) \). QED

3.10 (2 points) We will show that \( x \) is an odd integer if and only if \( x+1 \) is an even integer.

(\( \Rightarrow \)) Let \( x \) be an odd integer. Since \( x \) is odd there is an integer \( a \) with \( x = 2a + 1 \). Note that \( x + 1 = (2a + 1) + 1 = 2a + 2 = 2(a + 1) \). Therefore \( x + 1 \) is even.

(\( \Leftarrow \)) Now suppose that \( x + 1 \) is even. Therefore \( 2 \mid (x + 1) \) and hence there is an integer \( b \) with \( x + 1 = 2b \). Subtracting 1 from both sides of this equation, we get \( x = 2b - 1 = 2(b - 1) + 1 \) and so \( x \) is odd. QED

3.11 (2 points) Let \( x \) be an integer. We will show that \( 0 \mid x \) if and only if \( x = 0 \).

(\( \Rightarrow \)) Suppose \( 0 \mid x \). Since 0 is a divisor of \( x \) there is an integer \( a \) with \( x = 0 \cdot a \). Therefore \( x = 0 \cdot a = 0 \).

(\( \Leftarrow \)) Because \( 0 \cdot 1 = 0 \), we know that \( 0 \mid 0 \). QED

3.13 (2 points) Let \( x \) be an integer. We will show that \( x \) is odd if and only if \( x \) is the sum of two consecutive integers.
(⇒) Suppose $x$ is odd. By definition, there is an integer $a$ with $x = 2a + 1 = a + (a + 1)$. It follows that $x$ is the sum of two consecutive integers, namely $a$ and $a + 1$.

(⇐) Now suppose that $x$ is the sum of two consecutive numbers, say $b$ and $b + 1$. Then $x = b + (b + 1) = 2b + 1$, showing that $x$ is odd. QED