MthSc 119, Assignment 4 — Model Solutions

4.2 (1 point) A counterexample is $a = 6$ and $b = 0$. Then $6|0$ is true but $6 \leq 0$ is false.

4.3 (1 point) Let $a = 12$, $b = 4$, $c = 6$. Then $a|(bc)$ is true since $12|24$. However, neither $a|b$ nor $a|c$ is true: both $12|4$ and $12|6$ are false.

4.5 (2 points) Evaluating the polynomial for $n = 1, 2, \ldots, 10$ gives the values $43, 47, 53, 61, 71, 83, 97, 113, 131, 151$ all of which are prime. To disprove the statement "If n is a positive integer, then $n^2 + n + 41$ is prime," all we need is to find one exception. All the values $n = 1, 2, \ldots, 39$ give primes, but $n = 40$ gives $40^2 + 40 + 41 = 1681 = 41 \cdot 41$, which is not prime.

4.9 (1 point) For the statement "The integer a is composite if and only if a has two different prime factors" to be true both (a) "If the integer a is composite then a has two different prime factors" and (b) "If a has two different prime factors, then a is composite" must be true. However, statement (a) is not true: a counterexample is $a = 4$ which is composite but has only the prime 2 as a factor.