

MthSc 119, Section 5 – Model Solutions

5.1 (2.5 points) (a) FALSE. (b) TRUE. (c) FALSE. (d) FALSE. (e) TRUE.

5.3 (2 points) We make a table for $(x \wedge y) \vee (x \wedge \neg y)$ and compare to x .

x	y	$x \wedge y$	$x \wedge \neg y$	$(x \wedge y) \vee (x \wedge \neg y)$	x
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	F	F	F	F

Because the columns for $(x \wedge y) \vee (x \wedge \neg y)$ and for x are identical, we conclude that the two statements are logically equivalent.

5.6 (2 points) We make a truth table for $x \leftrightarrow y$ and $(x \rightarrow y) \wedge (y \rightarrow x)$.

x	y	$x \leftrightarrow y$	$x \rightarrow y$	$y \rightarrow x$	$(x \rightarrow y) \wedge (y \rightarrow x)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Because the columns for $x \leftrightarrow y$ and $(x \rightarrow y) \wedge (y \rightarrow x)$ are identical, we conclude that these statements are logically equivalent.

5.8 (2.5 points) We make a truth table for $(x \vee y) \rightarrow z$ and $(x \rightarrow z) \wedge (y \rightarrow z)$ and show that the corresponding columns are identical, letting us conclude that the two statements are identical.

x	y	z	$x \vee y$	$(x \vee y) \rightarrow z$	$x \rightarrow z$	$y \rightarrow z$	$(x \rightarrow z) \wedge (y \rightarrow z)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

5.11ab For each of these problems the last column should contain the entry TRUE in every row.

(a) (2 points)

x	y	$x \vee y$	$x \vee \neg y$	$(x \vee y) \vee (x \vee \neg y)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

(b) (2 points)

x	y	$x \rightarrow y$	$x \wedge (x \rightarrow y)$	$(x \wedge (x \rightarrow y)) \rightarrow y$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

5.15bc (b) (2 points) We make a truth table for the two statements in question, showing that the resulting columns are identical.

x	y	$x \underline{\vee} y$	$x \wedge \neg y$	$(\neg x) \wedge y$	$(x \wedge \neg y) \vee ((\neg x) \wedge y)$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

(c) (2 points) We show that the two statements in questions are identical via the following truth table:

x	y	$x \underline{\vee} y$	$x \vee y$	$\neg(x \wedge y)$	$(x \vee y) \wedge (\neg(x \wedge y))$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	F	T	F

(d) (1 point) We call $\underline{\vee}$ *exclusive* or because it captures the exclusive nature of the English *or*: you may have option A or B, *but not both*. In the same way $x \underline{\vee} y$ when x or y *but not both* are TRUE.