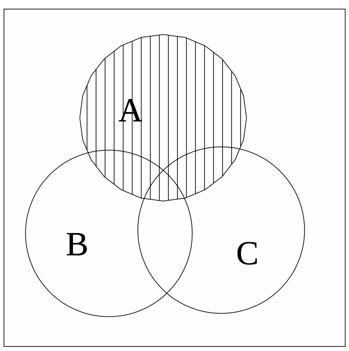
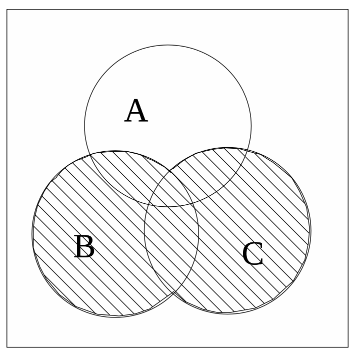
MthSc 119, section 10

10.1 (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$.
(b) $A \cap B = \{4, 5\}$.
(c) $A - B = \{1, 2, 3\}$.
(d) $B - A = \{6, 7\}$.
(e) $A \triangle B = \{1, 2, 3, 6, 7\}$.
(f) $A \times B = \{(1, 4), (1, 5), (1, 6), (1, 7), (2, 4), (2, 5), (2, 6), (2, 7), (3, 4), (3, 5), (3, 6), (3, 7), (4, 4), (4, 5), (4, 6), (4, 7), (5, 4), (5, 5), (5, 6), (5, 7)\}$.
(g) $B \times A = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (7, 1), (7, 2), (7, 3), (7, 4), (7, 5)\}$.

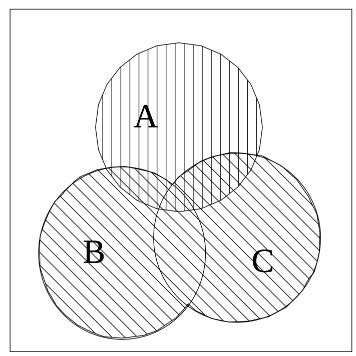
10.3 Here is a Venn diagram for the other distributive property:



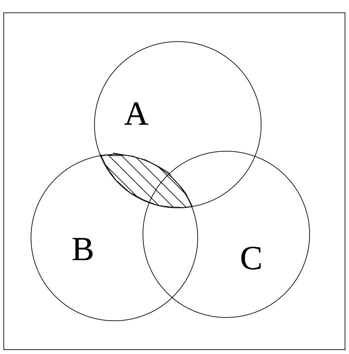
A



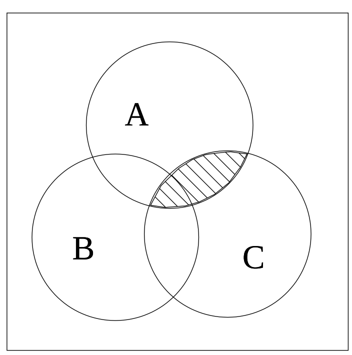
B U C



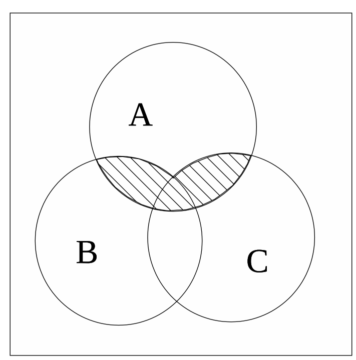
A∩(B U C)



A∩ B



A∩C



(A∩ B) U (A∩ C)

10.5 This is false. Let $A = \{1,2,3\}$, $B = \{3,4,5\}$, and $C = \{1,5\}$. Notice that $A \cap B \cap C = \emptyset$, so these sets satisfy the hypothesis of the statement. But $|A \cup B \cup C| = 5$, while $|A| + |B| + |C| = 8$.
10.8 This is false. Here is a counter-example: let $A = \{1,2,3\}$ and $B = \{3,4,5\}$. Then $|A \triangle B| = 4$ but $|A| + |B| - |A \cap B| = 3 + 3 - 1 = 5$.

10.14 We will show that $A - B = B - A$ iff $A = B$.
($\Rightarrow$) Suppose $A = B$. Then $A - B = \emptyset$ and $B - A = \emptyset$ (details were shown in class) and so $A - B = B - A$.
($\Leftarrow$) Suppose $A - B = B - A$. Note that this implies that $A - B = B - A = \emptyset$ an element cannot both be in A (as in $A - B$) and not in A (as in $A - B$) and since $A - B = \emptyset$, we have $A \subseteq B$, and since $B - A = \emptyset$ we have $B \subseteq A$. Therefore $A = B$. QED

10.16 (a) $A - (B - C) = (A - B) - C$ is false. A counterexample is $A = \{1,3\}$, $B = \{3,4\}$ and $C = \{3,5\}$.
(b) $(A - B) - C = (A - C) - B$ is true. Here is a proof:

$$\begin{aligned}
(A - B) - C &= \{x : x \in (A - B) \text{ and } x \notin C\} \\
&= \{x : x \in A \text{ and } x \notin B \text{ and } x \notin C\} \\
&= \{x : x \in A \text{ and } x \notin C \text{ and } x \notin B\} \\
&= \{x : (x \in A \text{ and } x \notin C) \text{ and } x \notin B\} \\
&= \{x : (x \in A - C) \text{ and } x \notin B\} \\
&= \{x : x \in (A - C) - B\} \\
&= (A - C) - B.
\end{aligned}$$

(c) $(A \cup B) - C = (A - C) \cap (B - C)$ is false. A counterexample is $A = \{1,2\}$, $B = \{2,3\}$ and $C = \{4\}$.
(d) "If $A = B - C$ then $B = A \cup C$" is false. A counterexample is $A = \{1\}$, $B = \{1,2\}$ and $C = \{2,3\}$.
(f) $|A - B| = |A| - |B|$ is false. A counterexample is $A = \{1,2\}$ and $B = \{2,3,4\}$.

10.21 See solutions for the same problem on quiz4.