Section 20

20.1 (a) f is a function. $\operatorname{dom} f = \{1, 3\}$. $\operatorname{im} f = \{2, 4\}$. f is one-to-one. $f^{-1} = \{(2, 1), (4, 3)\}$.

(b) f is a function. $\operatorname{dom} f = \mathbb{Z}$. $\operatorname{im} f$ is the set of all even integers. f is one-to-one and $f^{-1} = \{(x, y) : x, y \in \mathbb{Z}, 2y = x\}$.

(c) f is a function. $\operatorname{dom} f = \mathbb{Z}$. $\operatorname{im} f = \mathbb{Z}$. f is one-to-one. $f^{-1} = f$.

(d) f is not a function since $(0, 1), (0, 2) \in f$.

(e) f is a function. $\operatorname{dom} f = \mathbb{Z}$. $\operatorname{im} f$ is the set of all perfect squares. f is not one-to-one since $f(2) = f(-2)$ but $2 \neq -2$.

(g) f is not a function since $(0, 1), (0, -1) \in f$.

20.4 $\{(1, 3), (2, 3)\}$ is neither onto nor one-to-one.
$\{(1, 3), (2, 4)\}$ is both onto and one-to-one.
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20.6 (a) $f = \{(1, 5), (2, 5), (3, 6), (3, 7)\}$.
(b) $f = \{(1, 5), (2, 5), (3, 6), (4, 6)\}$.
(c) $f = \{(1, 5), (2, 5), (3, 6), (4, 7)\}$.

20.9 (a) f is one-to-one. Assume that $f(a) = f(b)$. Then $2a = 2b$ and dividing both sides of the equation by 2 we get $a = b$. Therefore f is one-to-one.

f is not onto. Suppose for the sake of contradiction that $f(x) = 5$ for some $x \in \mathbb{Z}$. Then $5 = 2x$ and dividing both sides by 2 we get $x = 1\frac{1}{2}$, which is not an integer. $\Rightarrow\Leftarrow$ Therefore f is not onto.

(b) f is one-to-one. Assume that $f(a) = f(b)$. Then $10 + a = 10 + b$ and subtracting 10 from both sides we get $a = b$.

f is also onto. Let x be an arbitrary integer. Observe that $x - 10$ is also an integer and that $f(x - 10) = 10 + x - 10 = x$. Therefore f is onto.

(d) f is not one-to-one. For example, $f(2) = \frac{2}{2} = 1$ yet also $f(3) = \frac{3-1}{2} = \frac{2}{2} = 1$. Thus $f(2) = f(3)$ with $2 \neq 3$, so f is not not one-to-one.