MthSc 119, section 8 – INCLUDING those due 2/9/05

8.1 abcd
(a) $\{0, 3, 6, 9\}$
(b) $\{2\}$
(c) $\{-2, 2\}$
(d) $\{\}$

8.2 (a) $\{x \in \mathbb{Z} : |x| \leq 10\} = \{-10, -9, -8, \ldots, -1, 0, 1, \ldots, 8, 9, 10\}$, which has cardinality 21.
(b) $\{x \in \mathbb{Z} : 1 \leq x^2 \leq 2\} = \{-1, 1\}$, which has cardinality 2.
(c) $\{x \in \mathbb{Z} : x \in \emptyset\} = \emptyset$ since there is *no* integer contained in $\emptyset$. Thus the cardinality of this set is 0.
(f) $2^{\{1,2,3\}}$ is the power set of $\{1, 2, 3\}$ which has cardinality $2^3 = 8$. $2^8 = 256$.
(g) $\{x \in 2^{\{1,2,3,4\}} : |x| = 1\} = \{\{1\}, \{2\}, \{3\}, \{4\}\}$, which has cardinality 4.

8.3 (a) $2 \in \{1, 2, 3\}$.
(b) $\{2\} \subseteq \{1, 2, 3\}$.
(c) $\{2\} \in \{\{1\}, \{2\}, \{3\}\}$.
(d) $\emptyset \subseteq \{1, 2, 3\}$.
(e) $\mathbb{N} \subseteq \mathbb{Z}$.
(f) $\{2\} \subseteq \mathbb{Z}$.
(g) $\{2\} \in 2^{\mathbb{Z}}$.

8.5 (2 points) Let $x \in A$. Thus x is an integer with $4|x$. It follows that for some integer a, $x = 4a = 2(2a)$, showing that $2|x$. Hence $x \in B$. QED

8.7 Let $x \in C$. Thus, by the definition of C, we know that $x|12$. By the definition of divisible it follows that $12 = xa$ for some integer a. Observe that $36 = 3 \times 12 = 3(xa) = x(3a)$, showing that $x|36$. Hence $x \in D$. QED

8.8 ($\Rightarrow$) Suppose $C \subseteq D$. Note that $c \in C$ and thus $c \in D$. Since $c \in D$ we have $c|d$.

($\Leftarrow$) On the other hand, suppose $c|d$. To show $C \subseteq D$ let $x \in C$. Thus $x|c$. We have $c|d$, so by Proposition 3.3, we have $x|d$ and so $x \in D$. Therefore $C \subseteq D$. QED

8.9 If $x = \emptyset$ then $x \subseteq \{x\}$ because $\emptyset$ is a subset of any set.