MthSc 119, section 9 – INCLUDING those due 2/9/05

9.1 (a) $\forall x \in \mathbb{Z}, x$ is prime.
(b) $\exists x \in \mathbb{Z}, x$ is not prime and x is not composite.
(c) $\exists x \in \mathbb{Z}, x^2 = 2$.
(d) $\forall x \in \mathbb{Z}, 5|x$.
(e) $\exists x \in \mathbb{Z}, 7|x$.
(g) $\forall x \in \mathbb{Z} \; \exists y \in \mathbb{Z}, xy = 1$.
(h) $\exists x \in \mathbb{Z} \; \exists y \in \mathbb{Z}, x/y = 10$.
(i) $\exists x \in \mathbb{Z} \; \forall y \in \mathbb{Z}, xy = 0$.
(j) $\forall x \in \mathbb{Z} \; \exists y \in \mathbb{Z}, y > x$.

9.2 (a) $\exists x \in \mathbb{Z}, x$ is not prime. "There is an integer that is not prime."
(b) $\forall x \in \mathbb{Z}, x$ is prime or x is composite. "Every integer is either prime or composite."
(c) $\forall x \in \mathbb{Z}, x^2 \neq 2$. "No integer has the square 2."
(d) $\exists x \in \mathbb{Z}, 5$ does not divide x. "Some integer is not divisible by 5."
(e) $\forall x \in \mathbb{Z}, 7$ does not divide x. "No integer is divisible by 7."
(g) $\exists x \in \mathbb{Z} \; \forall y \in \mathbb{Z}, xy \neq 1$. "There is an integer x such that multiplying it by any integer y never gives 1."

9.5 (a) $\exists x \in \mathbb{Z}, x \geq 0$. "There is some integer that is nonnegative."
(b) $\forall x \in \mathbb{Z}, x \neq x + 1$. "No integer x satisfies $x = x + 1$."
(d) $\exists x \in \mathbb{N}, x + x \neq 2x$. "There is a nonnegative integer x for which $x + x$ does not equal $2x$."
(g) $\exists x \in \mathbb{Z} \; \forall y \in \mathbb{Z}, x + y \neq 0$. "There is some integer x such that when any integer y is added to x, the result is never equal to zero."